Iterative methods for convex optimization problems

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Abstract. Let $U : X \to X$, where X is a convex subset of a Hilbert space H, be a an operator with nonempty $\operatorname{Fix} U = \{z \in X : Ux = x\}$. In our lectures we deal with iterative methods for finding a fixed point of U. We will focus on quasi-nonexpansive operators, i.e. operators U which satisfy

$$\|Ux - z\| \leq \|x - z\|$$

for all $z \in \operatorname{Fix} U$. The iterative methods presented in the lecture can be applied in various convex optimization problems, e.g. in:

- the convex feasibility problem (CFP):

  $$\text{find } x \in C = \bigcap_{i \in I} C_i$$

  if such point exists, where $C_i \subset H$, $i \in I = \{1, ..., m\}$, are convex, closed subsets of a Hilbert space H,

- the split feasibility problem (SFP):

  $$\text{find } x \in C \text{ such that } Ax \in D$$

  if such point exists, where $A : \mathbb{R}^n \to \mathbb{R}^m$ is an $m \times n$ matrix and $C \subset \mathbb{R}^n$, $D \subset \mathbb{R}^m$ are closed and convex subsets.

- the convex minimization problem

  $$\begin{array}{rc} \text{minimize} & f(x) \\ \text{subject to} & x \in D \end{array}$$

  where $f : \mathbb{R}^n \to \mathbb{R}$ is a convex function and $D \subset \mathbb{R}^n$ is a closed and convex subset.

We introduce some subclasses of the class of quasi-nonexpansive operators and give properties of these subclasses. We present several iterative methods for finding fixed points of the considered operators. We also show how to apply the presented methods to solving the considered problems. We prove the convergence of sequences generated by these methods. The methods can be applied in several large scale optimization problems, e.g. in the computerized tomography, in the intensity modulated radiation therapy, in signal processing and in many others.

References

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[SY98] H. Stark, Y. Yang, *Vector Space Projections. A Numerical Approach to Signal and Image Processing, Neural Nets and Optics*, John Wiley&Sons, Inc, New York, 1998.