M A T H E M A T I C S

STUDIES OF SECOND DEGREE

COURSE DIRECTORY

Since 2013/2014

ALGORITHMIC METHODS

Course code: 11.0-WK-MAT-SD-MAL
Type of course: optional
Language of instruction: English/Polish
Director of studies: dr Florian Fabiś
Name of lecturer: dr Florian Fabiś,
mgr Katarzyna Jesse-Józefczyk

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	15	1	III	Exam	
Laboratory	30	2		Grade	

COURSE AIM:

Extensive knowledge of algorithms' constructing and analysis. The ability to implement typical algorithms in practice and also the skills in adapting and modifying of those in extraordinary situations

ENTRY REQUIREMENTS:

Gaining of competences in computer structured programming. Basic course in algorithms and data structured.

COURSE CONTENTS:

Lecture

1. **NP – complete problems**. (2 h)
2. **Approximation algorithms.** Optimization and decision problems. Optimum and approximate solutions. Absolute performance guarantee and relative performance guarantee of approximation algorithm. Approximation schemes: PTAS, FPTAS. (3 h)
3. **Some approximation algorithms.** Vertex Cover, Set Cover, Bin Packing, Knapsack, Multiprocessor Scheduling, Graph Coloring, Traveling Salesman. (4 h)
4. **Algorithmic methods.** Greedy algorithms. Backtracking algorithms. Branch-and-Bound (BB) method. Dynamic programming. Genetic algorithms. Probabilistic algorithms. (6 h)

Laboratory

1. Generating random number. Generating random graphs. (2 h)
2. Selected combinatorial algorithms for practical applications (4 h)
3. Approximation algorithms. (8 h)
4. Testing of algorithms that use selected algorithmic methods. (6 h)
5. Probabilistic algorithms. (4 h)
6. Selected algorithms with numbers. (6 h)

TEACHING METHODS:

Lecture: problem lecture.

Laboratory: laboratory exercises in computer lab – implementation and testing of selected algorithms.

Each student is supposed to realize three projects during the semester. Each project will consist in implementation of the selected algorithm to solve a concrete practical task, writing a program for it, testing it and presenting a documentation in accordance with the assigned specification. On one out of the three projects the students will work in 2-3 person groups. Furthermore the students will test other algorithms.

LEARNING OUTCOMES:

Student has knowledge of advanced methods of constructing efficient algorithms. [K_W11++]
Student knows the basic approximation algorithms and can implement them in programs [K_W11++], [K_U19++], [K_U20++]
Student knows the concept of probabilistic algorithms and can give examples of their use. [K_W11++]
Student knows the selected algorithms with numbers. [K_W11++]
Student is able to work in project team [K_K03++]

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Lecture. Written examination verifying the education outcome in area of knowledge and skills.

Laboratory. Final grade is granted based on number of points received during studies. Points are received for written tests, active participation in classes and completed project.

Final course grade consists of laboratory classes' grade (50%) and examination grade (50%). Positive grade from laboratory classes is the necessary condition for participation in examination. The positive grade from examination is the necessary condition for course completion.

STUDENT WORKLOAD:

Contact hours

- Participation in lectures: 15*1 h = 15 h
- Participation in laboratory studies : 15*2 h = 30 h
- Consultations: = 8 h
- Participation in the exam:1*2 h = 2 h

Total: 55 h (3 ECTS)

Independent work

- Preparation for laboratory exercises: 15*3 h = 45 h
- Finishing in house exercise laboratory: 15*2 h = 30 h
- Exam preparation: 20 h

Total: 95 h (3 ECTS)

Total for the course: 150 h (6 ECTS)

RECOMMENDED READING:

1. Aho A., Hopcroft J.E., Ullman J.D.: Projektowanie i analiza algorytmów komputerowych, PWN, Warszawa 1983.
2. Błażewicz J. : Złożoność obliczeniowa problemów kombinatorycznych, WNT, Warszawa 1988.
3. Cormen T.H., Leiserson C.E., Rivest R.L., Wprowadzenie do algorytmów, WNT, Warszawa 1997.
4. Vazirani V. V. : Algorytmy aproksymacyjne, WNT, 2004.

OPTIONAL READING:

1. Aho A., Hopcroft J.E., Ullman J.D., : The Design and Analysis of Computer Algorithms.
2. T.H. Cormen, Ch.E. Leiserson, R.L. Rivest: Introduction to Algorithms, MIT Press, 2001.
3. Knuth D.E.: The Art of Computer Programming.
4. Vazarni V. V. : Approximation Algorithms, Springer, 2003.

COMBINATORIAL ANALYSIS

Course code: 11.1-WK-MAT-SD-AK
Type of course: optional
Language of instruction: English/Polish
Director of studies: dr Magdalena Łysakowska
Name of lecturer: dr Magdalena Łysakowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					5
Lecture	30	2	II or IV	Exam	
Class	30	2		Grade	

COURSE AIM:

Introducing students to basic definitions, theorems and methods of combinatorial analysis and examples of applications of them.

ENTRY REQUIREMENTS:

Completed courses of mathematical analysis, linear algebra and discrete mathematics.

COURSE CONTENTS:

Lecture

1. The binomial coefficients (2 h)
2. Rook polynomials (2 h)
3. Latin squares (2 h)
4. Van der Waerden's Theorem, Schur's Theorem (2 h)
5. Map-colourings, Four – Colour Theorem (3 h)
6. Minimax theorems (4 h)
7. Combinatorial designs (2 h)
8. Perfect codes, Hadamard's matrices (5 h)
9. Sperner's Lemma (3 h)
10. Minkowski's Theorem, Radon's Theorem, Helly's Theorem, Tverberg's Theorem (5 h)

Class

1. Proving combinatorial identities (2 h)
2. Applications of rook polynomials (3 h)
3. Making latin squares; proving properties of latin squares (3 h)
4. Applications of van der Waerden's and Schur's Theorems (2 h)

Test (2 h)

5. Applications of Four - Colour Theorem and minimax theorems (4 h)

6. Proving properties of combinatorial designs; applications of combinatorial designs (3 h)
7. Constructing of perfect codes (3 h)
8. Applications of Sperner's Lemma and basic theorems of combinatorial geometry (6 h)

Test (2 h)

TEACHING METHODS:

Traditional lecture, discussion exercises, work in groups

LEARNING OUTCOMES:

1. A student is able to perform proofs of basic combinatorial identities (K_U10)
2. A student is able to apply root polynomials to solve practical exercises (K_U10)
3. A student knows König-Egarváry's, Menger's, Ford-Fulkerson's Theorems, Four – Colour Theorem and is able to apply them to solve practical exercises (K_W03)
4. A student is able to perform the proof of Fisher's Theorem, knows the definition and examples of finite projective planes, is able to point connections between combinatorial designs and projective planes (K_W04)
5. A student is able to use Hadamard's matrices and combinatorial designs to construct codes (K_U10)
6. A student knows Sperner's Lemma, Schur's Theorem, van der Waerden's Theorem, Minkowski's Theorem, Radon's Theorem, Helly's Theorem, Tverberg's Theorem, knows proofs of this theorems and examples of their applications (K_W03,K_W04)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Checking of preparedness of students and their activity during exercise
2. Colloquiums with tasks of different difficulty, allowing to evaluate whether the students have achieved specified learning outcomes in minimal level
3. Written exam

The grade of the module is the arithmetic mean of the exercise grade and the exam grade. The prerequisite of the exam is to get a positive assessment of the exercise. The condition to obtain a positive evaluation of the module is the positive evaluation of the exam.

STUDENT WORKLOAD:

Lectures – 30 h

Exercise – 30 h

Consultation – 10 h

Self preparation for lectures – 10 h

Self preparation for exercise – 30 h

Self preparation for colloquiums – 10 h

Self preparation for the exam – 20 h

The combined student workload – 140 h (5 ECTS)

RECOMMENDED READING:

1. W. Lipski, W. Marek, *Analiza kombinatoryczna*, PWN, Warszawa,1986.
2. K. A. Rybnikow (red.), *Analiza kombinatoryczna w zadaniach*, PWN, Warszawa, 1988.
3. J. Matoušek, *Lectures on Discrete Geometry*, Springer, New York, 2002.

OPTIONAL READING:

1. Z. Palka, A. Ruciński, *Wykłady z kombinatoryki*, WNT, Warszawa, 1998.
2. R. L. Graham, D. E. Knuth, O. Patashnik, *Matematyka konkretna*, PWN, Warszawa, 2011.
3. V. Bryant, *Aspekty kombinatoryki*, WNT, Warszawa, 1997.

CONTROL THEORY 1

Course code:	11.1-WK-Mat-SD-TS1
Type of course:	optional
Language of instruction:	Polish/English
Director of studies:	prof. dr hab. Jerzy Motyl
Name of lecturer:	prof. dr hab. Jerzy Motyl, dr Joachim Syga, dr Maciej Niedziela

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	II or IV	Exam	
Class	30	2		Grade	

COURSE AIM:

After the course of “control theory 1” students should be able to solve themselves practical and theoretical problems on the topic of dynamical linear systems.

ENTRY REQUIREMENTS:

Linear algebra, differential equations

COURSE CONTENTS:

Lecture:

1. Dynamical systems – definitions and classification (4 h.).
2. Main theorem on the smooth system (2 h.).
3. Costs functional - problems of Meyer, Lagrange and i Bolza (2 h.).
4. Differential types of controllability (2 h.).
5. Linear dynamical systems, fundamental matrix (2 h.).
6. Gram matrix, its properties and connections with global controllability (2 h.).
7. Theorems of Kalman’s type for discrete and continuous linear dynamical systems (4 h.).
8. Linear-quadratic problem (2 h.).
9. Properties of attainable set, emission zone and the set of attainable controls (2 h.).
10. Theorems on properties of the attainable set: convexity, boundedness, compactness (4 h.).
11. Extremal controls (2 h.).
12. Integral maximum rule (2 h.).

Class

1. Linear equations and their fundamental matrix different methods of solving (4h.).
2. Linear dynamical systems and „0-1” fundamental matrix (2 godz.).
3. Gram matrix solving and its connections with global controllability (2 h.).
4. Solving of global controllability of discrete and continuous linear dynamical systems by Kalman’s methods (6 h.).
5. Solving of linear-quadratic problem (4 h.).

6. Properties of attainable set, emission zone and the set of attainable controls (2 h.).
7. Examples of the nonexistence of optimal controls without convexity or compactness of attainable controls (2 h.).
8. Extremal controls for linear dynamical systems (4 h.).
9. Applicability of the integral maximum rule (2 h.).

TEACHING METHODS:

Conventional lecture; problem lecture
Auditorium exercises – solving standard problems enlightening the significance of the theory, exercises on applications, solving problems.

LEARNING OUTCOMES:

1. K_W04 has in-depth knowledge in the chosen field of theoretical mathematics or applied
2. K_U03 has the ability to validate evidence of formal building of proofs
3. K_U14 in the selected field can carry out evidence which, if necessary, also the tools from other departments of mathematics
4. K_K04 is able to formulate opinions on the basic issues of mathematical proofs

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Final exam and grade

STUDENT WORKLOAD:

Lectures - 30 h
Classes - 30 h
Tutoring – 15 h (Lectures - 5 h; Classes - 10 h)
Total: 75 h (3 ECTS)
Individual students` work
Preparing to lectures - 25 h
Preparing to classes - 35 h
Preparing to the exam - 40 h
Total: 100 h (4 ECTS)
Total hours and points per course 175 h (7 ECTS)

RECOMMENDED READING:

1. J. Zabczyk, Zarys matematycznej teorii sterowania, PWN, 1991
2. Z. Wyderka, Teoria sterowania optymalnego, skrypty Uniwersytetu Śląskiego nr 397, Katowice, 1987.

OPTIONAL READING:

1. S. Rolewicz, Analiza funkcjonalna i teoria sterowania, PWN, 1977.

CONTROL THEORY 2

Course code: 11.1-WK-Mat-SD-TS2
Type of course: optional
Language of instruction: Polish/English
Director of studies: prof. dr hab. Jerzy Motyl
Name of lecturer: prof. dr hab. Jerzy Motyl

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	III	Exam	
Class	30	2		Grade	

COURSE AIM:

After the course of "control theory 2" students should be able to solve themselves practical and theoretical problems on the topic of dynamical nonlinear systems and set valued functions.

ENTRY REQUIREMENTS:

Measure theory, Lebesgue integral, control theory 1

COURSE CONTENTS:

Lecture:

1. Problems of optimal control theory
2. Controllability and properties of attainable sets of dynamical control systems
3. Support functions and their properties
4. Hausdorff metric and continuity of set valued functions
5. Nonlinear control systems as differential inclusions
6. Continuity and measurability of set valued functions
7. Selections problems: minimal, Czebyszev's, barycentric and Steiner selections
8. Selections Theorems of Michael and Kuratowski Ryll-Nardzewski
9. Fixed point Kakutani theorem
10. Filipov's theorem
11. Aumann's integral and its properties
12. Existence of solutions of differential inclusions and its connections with control problems
13. Viability problem

Class

1. Live problems leading to optimal control theory
2. Properties of attainable set and the set of attainable controls
3. Support functions and their properties
4. Hausdorff metric and continuity of set valued functions
5. Upper and lower semicontinuous set valued functions
6. Minimal, Tschebyshev's, barycentric and Steiner selections
7. Properties of contingent cones and viability problem

TEACHING METHODS:

Conventional lecture; problem lecture
Auditorium exercises – solving standard problems enlightening the significance of the theory, exercises on applications, solving problems.

LEARNING OUTCOMES:

1. K_W06 links selected issues knows areas of theoretical and applied mathematics with other departments
2. 2 K_U01 has the skills to construct mathematical understandings: command statement, and debunking the hypotheses through the construction and selection of counterexamples.
3. K_U13 place, advanced level and covering mathematics and apply contemporary present in speech and in writing, at least one of the selected branches of Mathematics: mathematical analysis and functional analysis, theory of differential equations and dynamical systems
4. K_K01 knows the limitations of his own knowledge and understands the need for further training.
5. K_K04 is able to formulate opinions on the basic issues of mathematical proofs

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Final exam and grade

STUDENT WORKLOAD:

Lectures - 30 h
Classes - 30 h
Tutoring – 15 h (Lectures - 5 h; Classes - 10 h)
Total: 75 h (3 ECTS)
Individual students` work
Preparing to lectures - 35 h
Preparing to classes - 40 h
Preparing to the exam - 35 h
Total: 110 h (4 ECTS)
Total hours and points per course 185 h (7 ECTS)

RECOMMENDED READING:

1. M. Kisielewicz, Differential Inclusions and Optimal Control, PWN – Kluwer Acad. Publ. 1991.
2. J.P. Aubin, A. Cellina, Differential Inclusions, Springer Verlag 1984.
3. Z. Wyderka, Teoria sterowania optymalnego, skrypty Uniwersytetu Śląskiego nr 397, Katowice 1987.

OPTIONAL READING:

1. S. Rolewicz, Analiza funkcjonalna i teoria sterowania, PWN 1977.
2. J. Zabczyk, Zarys matematycznej teorii sterowania, PWN 1991.

DIFFERENTIAL EQUATIONS

Course code: 11.1-WK-MAT-SD-RR
Type of course: optional
Language of instruction: English/Polish
Director of studies: dr Tomasz Małolepszy
Name of lecturer: dr Tomasz Małolepszy

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					8
Lecture	30	2	III	Exam	
Class	15	1		Grade	
Laboratory	15	1		Grade	

COURSE AIM:

The main aim of this course is to familiarize students with the theory of ordinary differential equations, with particular emphasis on the qualitative theory.

ENTRY REQUIREMENTS:

Mathematical Analysis 1 and 2, Linear Algebra 1 and 2, Mathematical Software.

COURSE CONTENTS:

1. First-order ordinary differential equations.
   Basic concepts. Geometrical interpretation of ODE. ODE integrable by quadratures.
2. Existence and uniqueness of local solutions of the initial problems for ODE.
   Cauchy problem for ODE. Existential theorems (Picard-Lindelöf theorem, Peano theorem). Extension of solutions of the initial problems for ODE. Dependence of the solution to Cauchy problem on initial conditions and the right-hand side of the equation.
3. High-order ordinary differential equations.
   Types of equations reducible to first-order ordinary differential equations. Linear second-order differential equations. Sturm-Liouville boundary problem.
4. Dynamical interpretation of systems of ODE.
   Autonomous systems. Phase trajectories and phase portraits. Flows and orbits. First integrals.
5. Systems of linear ordinary differential equations.
   Methods of solving of homogeneous and inhomogeneous systems of linear equations. Classification and stability of critical points of systems of linear ODE in the plane. Phase portraits.
6. Systems of nonlinear ordinary differential equations.
   Local phase portraits. Linearization, Grobman-Hartman theorem. Classification and stability of critical points of systems of nonlinear ODE in the plane. Global phase portraits.

7. Periodic orbits and limits cycles.
   Limits sets. Poincaré-Bendixson theorem.
8. Elements of the stability theory.
   Lyapunov stability. Hurwitz theorem. Lyapunov function and fundamental stability theorems.
9. Bifurcations and chaos.
   Hopf bifurcation. The Lorenz model.
10. Some differential models in physics, biology, medicine and economics.
    Van der Pol oscillator. Lotka-Volterra systems. Epidemiological models. May model. Solow model and economic cycle models.

Classes.

Solving of problems related to contents of lectures with particular emphasis on practical applications of learned concepts.

Laboratory.

Solving of problems related to ODE by means of mathematical software.

TEACHING METHODS:

Traditional lectures; classes with the lists of exercises to solve by students; computer lab.

LEARNING OUTCOMES:

Student is able:

1. to use some qualitative methods to examine ODEs, (K_U04+, K_U06++, K_K01+)
2. to interpret systems of ODEs in terms of dynamical systems, (K_U06++)
3. to solve ODEs describing some basic physical phenomena by means of tools used in computer science. (K_W11+, K_U15+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Class and Laboratory: learning outcomes will be verified through two tests consisted of exercises of different degree of difficulty. A grade determined by the sum of points from these two tests is a basis of assessment.

Lecture: final exam. A grade determined by the sum of points from that exam is a basis of assessment.

A grade from the course is consisted of the grade from laboratory (20%), the grade from classes (30%) and the grade from the final exam (50%). To take a final exam, students must receive a positive grade from classes. To attain a pass in the course students are required to pass the final exam.

STUDENT WORKLOAD:

Contact hours

Lectures - 30 hours.

Classes - 15 hours.

Laboratories - 15 hours.

Lectures’ consultation hours - 5 hours.

Classes’ consultation hours - 2.5 hours.

Laboratories’ consultation hours - 2.5 hours.

Total - 70 hours (3 ECTS).

Individual work

Preparation to lectures - 40 hours.

Preparation to classes - 30 hours.

Preparation to laboratories - 30 hours.

Preparation to the final exam - 30 hours.

Total - 130 hours (5 ECTS).

Total time needed for this course: 200 hours (8 ECTS).

RECOMMENDED READING:

1. A. Palczewski, *Równania różniczkowe zwyczajne*, WNT, Warszawa, 1999.
2. W. I. Arnold, *Równania różniczkowe zwyczajne*, PWN, Warszawa, 1975.
3. D. K. Arrowsmith, C.M. Place, *Ordinary differential equations, A qualitative approach with applications*, Chapman and Hall, London, 1982.
4. A. Pelczar, J. Szarski, *Wstęp do równań różniczkowych zwyczajnych*, PWN, Warszawa, 1987.
5. N. M. Matwiejew, *Metody całkowania równań różniczkowych zwyczajnych*, PWN, Warszawa, 1986.

OPTIONAL READING:

1. L. S. Pontriagin, *Równania różniczkowe zwyczajne*, PWN, Warszawa, 1964.
2. Ph. Hartman, *Ordinary Differential Equations*, Wiley, New York, 1964.

DIFFERENTIAL GEOMETRY

Course code: 11.1-WK-MAT-SD-GR
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Andrzej Kisielewicz
Name of lecturer: dr Andrzej Kisielewicz

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	I	Exam	
Class	30	2		Grade	

COURSE AIM:

Foundations of differential geometry

ENTRY REQUIREMENTS:

Differential calculus, linear algebra, topology

COURSE CONTENTS:

Local theory of curves

1. Parametrization of curves, parametrization by arclength
2. The length of a curve
3. The Frenet frame
4. The Frenet formulas
5. Curvature and torsion of a curve
6. Characterizations of selected curves by its curvature and torsion
7. Canonical form of a curve

Global theory of curves

1. The Fundamental Theorem of Curve Theory
2. The Crofton formula
3. The Fenchel theorem
4. The Schur theorem
5. The four vertex theorem
6. The isoperimetric inequality

Local theory of surfaces

1. Parametrization of surfaces
2. The first fundamental form
3. The surface area
4. The Gauss map
5. The shape operator
6. The second fundamental form
7. Gaussian and mean curvature
8. Theorema Egregium

Global theory
1. The Liebmann theorem
2. The fundamental theorem of surface theory
3. Minimal surfaces
4. Geodesics

Class

Local theory of curves
1. Parametrizations of selected curves.
2. Calculation of the length of selected curves
3. Calculation of the Frenet frame of selected curves
4. Applications of the Frenet formulas
5. Calculation of curvature and torsion of selected curves
6. Characterizations of selected curves by its curvature and torsion

Local theory of surfaces
1. Parametrization of a surface
2. Calculation of the first fundamental form of selected surfaces
3. Calculation of areas of selected surfaces
4. Calculation of the shape operator of selected surfaces
5. Calculation of the second fundamental form of selected surfaces
6. Calculation of Gaussian and mean curvature of selected surfaces
7. Calculation of geodesics of selected surfaces

TEACHING METHODS:

Lecture and disccusions

LEARNING OUTCOMES:

1. Students are able to find parametrizations of basics curves and surfaces (K_U06,K_U10)
2. Students are able to compute curvature and torsion of basics curves and Gaussian curvature of basic surfaces (K_U05,K_U11)
3. Students are able to compute Gaussian curvature of basic surfaces (K_U05,K_U11)
4. Students understand in what manner differential calculus is connected to geometry (K_U04,K_W09)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Exams and talks

STUDENT WORKLOAD:

1. Lecture-30 hours
2. Class-30 hours
3. Consultations-10 hours

Individual work
1. Lecture preparation -40 hours
2. Class preparation -40 hours
3. Exams preparation-30 hours

RECOMMENDED READING:

1. T. Shifrin, Differential Geometry: A First Course in Curves and Surfaces, 2007 (www.math.uga.edu/~**shifrin**/**Shifrin**DiffGeo.pdf)
2. J. Oprea, Geometria różniczkowa I jej zastosowania, PWN, Warszawa, 2002.

OPTIONAL READING:

1. H. Hopf, Differential Geometry in the Large, Springer, 1983.

DISCRETE MATHEMATICS AND MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Course code: 11.0-WK-MAT-SD-MDMPI
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr hab. Ewa Drgas-Burchardt
Name of lecturer: dr hab. Ewa Drgas-Burchardt
dr Elżbieta Sidorowicz

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	I	Exam	
Class	30	2		Grade	

COURSE AIM:

Differing and counting combinatorial objects. Using these capabilities, in order to estimate the size of the data and resources needed to solve the problem and study its complexity.

ENTRY REQUIREMENTS:

Completed vocational mathematical or technical studies

COURSE CONTENTS:

Lecture

1. Elements of combinatorics: counting methods for labeled and unlabeled combinatorial objects, Polya's Enumeration Theorem, extremal set theory (12 h).
2. Elements of graph theory: connectivity, matchings, Hall's Theorem, Hamiltonian cycles, vertex and edge colorings, planarity, extremal graph theory questions, Turans's Theorem, Ramsey's Theorem (6 h).
3. The probabilistic method of Erdös (6 h).
4. Elements of counting theory: transition functions, automata, Turing machines, formal languages, Church's assertions (6 h).

Class

1. Elements of combinatorics:
   a. combinatorial object recognition in the practical problems, the concept of functions operating on finite sets, which are free, injective, "on", decreasing, non-increasing, the usage of well-known formulas to count the identified objects (5 h),
   b. application of Inclusion-Exclusion Principle and Pigeonhole Principle, double counting of labeled combinatorial objects (5 h),
   c. application of Polya's Enumeration Theorem in order to count unlabeled combinatorial objects (4 h).

2. Elements of graph theory:
    a. recognition of notions of graph theory in practical tasks, application of well-known algorithms to solve these tasks (2 h),
    b. estimation of small Ramsey numbers, proving theorems on Ramsey graphs and numbers by usage of the classical techniques presented during the lectures (2 h).
3. The probabilistic method of Erdös: prooving the facts on the combinatorial structures by usage of the naive method, the expected value method and Local Lovás Lemma (6 h).
4. Elements of the theory of computation: constructing programs for Turing machines, computability test function (4 h).
5. Test completion (2h).

TEACHING METHODS:

Conversation lecture, traditional lecture, discussion exercises

LEARNING OUTCOMES:

1. A student is able to name and define the basic concepts of discrete mathematics and theory of computation (K_W01, K_W12).
2. A student is able to perform proofs in the field of discrete mathematics and theory of computation, in which, if necessary, he/she applies the concept of probability space or linear space (K_W06, K_U01, K_U14).
3. A student is able to use the methods of mathematical analysis, algebra and probability to test the convergence of sequences and series, to solve systems of linear equations, to axiomatic recognize of groups, to test the independence of events and random variables and study their characteristics in order to solve tasks in the field of discrete mathematics and theory calculations (K_W03).
4. A student is able to decide with which objects in the field of discrete mathematics and theory of computation, the solution of the practical problem can be identified (K_U13).
5. A student understands the significance of intellectual honesty, both in their own and in other people's activities; demonstrate ethical behavior (K_K03).

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Methods:
D - participation in the discussions during the course
P1 - essay
P2 - written exam
PU2 - oral exam
S - self-esteem

Assessment of individual classes:

1. Checking of preparedness of students and their activity during exercise (D, S).
2. Colloquium with the tasks of different difficulty, allowing to evaluate whether the students have achieved specified learning outcomes in terms of skills and competencies (P1).
3. Conversation during the lectures in order to verify the effects of higher levels of education in terms of knowledge and skills (D, S).
4. Written exam to verify the learning outcomes in terms of knowledge and skills (P2).
5. Oral exam, which allows complete student's written expression (PU2).

The grade of the module consists of the assessment exercise (50%), exam grade (P2 + PU2) (50%). The condition of the exam is to get a positive assessment of the exercises. The prerequisite to obtain a positive evaluation of the module is the positive evaluation of the exercise and the exam.

STUDENT WORKLOAD:

Activity	Student load
Participation in lectures	30 h
Participation in exercises	30 h
Self preparation for lectures	50 h
Independent problem solving	50 h
Consultation	15 h
The combined student workload	175 h

Number of ECTS credits allocated 7

RECOMMENDED READING:

1. W. Lipski, W. Marek, Analiza kombinatoryczna, PWN, Warszawa, 1989.
2. C.H. Papadimitriou, Złożoność obliczeniowa, WNT, Warszawa 2002 (seria Klasyka Informatyki).
3. Diesel, Grach Theory, Springer-Verlag, Heidelberg, Gradule Test In Mathematics, Vol. 173.
4. Kościelski, Teoria obliczeń, WUW, Wrocław, 1997.

OPTIONAL READING:

1. Z. Palka, A. Ruciński, Wykłady z kombinatoryki, cz. I, WNT, Warszawa, 1998.
2. W. Lipski, Kombinatoryka dla programistów, WNT, 2005.
3. K.A. Ross, Ch.R.B. Wright, Matematyka dyskretna, PWN, Warszawa, 1996.
4. R.J. Wilson, Wprowadzenie do teorii grafów, PWN, 1998.

MATHEMATICAL ECONOMICS 2

Course code: 11.1-WK-MAT-SD-EM2
Type of course: optional
Language of instruction: English /Polish
Director of studies: prof. dr. hab. Andrzej Nowak
Name of lecturer: prof. dr. hab. Andrzej Nowak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	IV	Exam	
Class	30	2		Grade	

COURSE AIM:

Knowledge of foundations and basic theorems of growth theory and dynamic systems in economics.

ENTRY REQUIREMENTS:

Mathematical economics 1, foundations of optimization theory, probability theory

COURSE CONTENTS:

Lectures:

I. Growth model:
1. Harold, Solow-Swan and Frankel Model. (2 hrs)
2. Ramsey Model. (2 hrs)
3. Value function, Bellman equations. (2 hrs)
4. Properties of value functions and optimal policies. (2 hrs)

II. Multisector growth model:
1. Ramsey model. (2 hrs)
2. Consumption and saving problem. (2 hrs)
3. Examples of optimal policy and value function. (4 hrs)

III. Cycles and chaos in growth models:
1. Examples of chaos. (2 hrs)
2. Existence of periodic orbits. (2 hrs)

IV. Stochastic growth models:
1. Formulation of the problem. (2 hrs)
2. Markov decision process. (4 hrs)

V. Neoclassical Brock-Mirman model:
1. Bellman equation. (2 hrs)
2. Existence of stationary distribution. (2 hrs)

Class:

I. Growth model:

1. Berge maximum theorem. (4 hrs)
2. Ramsey model. (2 hrs)
3. Value function, Bellman equation. (2 hrs)
4. Properties of value function and optimal policy. Analytic solution in Levhari-Mirman model. (2 hrs)

II. Multisector growth model:

1. Ramsey model. Examples. (2 hrs)
2. Consumption and saving model. Solving Simple problems. (2 hrs)
3. Examples of optymat policies and value functions. (4 hrs)

III. Cycles and chaos in growth models:

1. Examples of chaos. (2 hrs)
2. Existence of periodical orbits. Examples. (2 hrs)

IV. Stochastic growth models:

1. Formulation of the problem. (2 hrs)
2. Markov decision process. Examples. (4 hrs)

V. Colloquium: (4 hrs)

TEACHING METHODS:

Lectures and classes

LEARNING OUTCOMES:

Student

1. understands significance of mathematical methods in economics (K_W04),
2. knows basic fixed point theorems and knows their application in economics (K_W01),
3. recognizes Bellman rule in dynamic optimization (K_U16),
4. recognizes economic growth models (K_U16),
5. knows Basic notions in the area of Markov decision processes (K_W04),
6. is able to to verify stability in simple dynamic economic model (K_U14),
7. graduates and understands the need for lifelong education (K_K01, K_U19).

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Evaluation of individual exercises, final exam and grade.

STUDENT WORKLOAD:

Contact hours

Lecture – 30 hrs

Class – 30 hrs

Office hours – 15 hrs (5 hrs for lecture and 10 for class)

Total 75 hrs (3 ECTS)

Individual work

Preparing to lecture – 25 hrs

Preparing to class – 35 hrs

Preparing to exam – 40 hrs

Total: 100 hrs (4 ECTS)

Total hours and points per course: 175 hrs (7 ECTS)

RECOMMENDED READING:

1. Le Van, C., Dana R-N, Dynamic Programming In Economics,Kluwer Acad. Dordrecht,2003.
2. Bhattacharya R. Majumdar M. Random Dynamical Systems Theory and Applications, Cambridge Univ.Press. 2007.

OPTIONAL READING:

1. Tokarski, T. Ekonomia matematyczna. Modele mikroekonomiczne, Polskie Wydawnictwo Ekonomiczne, Warszawa, 2011.
2. Tokarski T. Ekonomia matematyczna. Modele makroekonomiczne, Polskie Wydawnictwo Ekonomiczne, Warszawa, 2011.

MATHEMATICAL PROGRAMMING

Course code: 11.0-WK-MAT-SD-PM
Type of course: optional
Language of instruction: English
Director of studies: prof. Andrzej Cegielski
Name of lecturer: prof. Andrzej Cegielski, dr. Robert Dylewski, dr. Tomasz Małolepszy

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					10
Lecture	30	2	II or IV	Examination	
Class	30	2		Written test	
Laboratory	30	2		Written test	

COURSE AIM:

The lecture should give a knowledge on methods for constrained minimization, in particular on methods for linear programming and quadratic programming. Furthermore, the lecture contains foundations of multicriterial and nondifferentiable minimization. In the laboratory the students apply an appropriate software.

ENTRY REQUIREMENTS:

Linear algebra 1 and 2, mathematical analysis 1 and 2, foundations of optimization

COURSE CONTENTS:

1. **Linear programming.** Linear programming (LP) problems and problems which can be reduced to LP. Graphic method. Simplex algorithm, I and II phase. Duality in LP and the dual simplex algorithm.
2. **Quadratic programming**. Methods for equality constraints and for inequality constraints, active set method.
3. **Constrained minimization methods**. Reduction to unconstrained minimization: penalty function and barrier function. SQP-method.
4. **Linear multi-criterial programming.** Pareto-optimal solution. Optimal solution with respect to a meta-criterion.
5. **Convex nondifferentiable minimization.** Fejer monotonicity. Optimality conditions. Subgradient projection method.

TEACHING METHODS:

Traditional lecture, classes with exercises, laboratory with application of appropriate software

LEARNING OUTCOMES:

Student

- Can construct mathematical models for simple optimization problems (K_U25++)
- Knows and understands the graphic method for two-dimensional optimization problems (K_U10+)
- Knows basic methods for multi-criterial optimization (K_U10+)
- Is able to apply basic minimization methods for simple constrained minimization problems (K_W11++, K_U10+)
- Is able to apply subgradient projection method to simple convex nondifferentiable problems (K_W11+)
- Knows and applies an appropriate software to symbolic calculus and to simple optimization problems (K_U15++, K_U13++)
- Understands the necessity of an application of mathematical methods in practical problems (K_K04+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Checking the activity of the student
2. Written tests
3. Checking the ability of application of an appropriate software
4. Written examination

The final grade consists of the classes grade (30%), the lab's grade (30%) and the examination's grade (40%)

STUDENT WORKLOAD:

Contact hours

- Participation in lectures: 30h
- Participation in exercises: 30h
- Participation in laboratory: 30h
- Consultations: 10h (lecture), 20h (laboratory)

Independent work

- Preparation for lectures: 30h
- Preparation for laboratory: 30h
- Preparation for exercises: 30h
- Preparation for examination: 40 h

Total: 250h (10 ECTS)

RECOMMENDED READING:

1. A. Cegielski, Podstawy optymalizacji, skrypt do wykładu
2. W. Findeisen, J. Szymanowski, A. Wierzbicki, Teoria i metody obliczeniowe optymalizacji, PWN, Warszawa, 1980.
3. Z. Galas, I. Nykowski (red.), Zbiór zadań z programowania matematycznego, część I, II, PWN, Warszawa, 1986, 1988.
4. W. Grabowski, Programowanie matematyczne, PWE, Warszawa, 1980.
5. A. Cegielski, Programowanie matematyczne - część 1 - Programowanie liniowe, Uniwersytet Zielonogórski, Zielona Góra, 2002.
6. Badania operacyjne (red. W. Sikora), PWE, Warszawa, 2008.

OPTIONAL READING:

1. M. S. Bazaraa, H. D. Sherali, C. M. Shetty, Nonlinear Programming, Third Edition, J. Wiley&Sons, Hoboken, NJ, 2006
2. D. P. Bertsekas, Nonlinear Programming, Athena Scientific, Belmont, MA, 1995
3. J.E. Dennis, R.B. Schnabel, Numerical Methods for Unconstrained Optimization and Nonlinear Equations, SIAM, Philadelphia 1996.
4. R. Fletcher, Practical Methods of Optimization, Vol I, Vol. II, John Willey, Chichester, 1980, 1981.
5. M. Brdyś, A. Ruszczyński, Metody optymalizacji w zadaniach, WNT, Warszawa, 1985.
6. J. Stadnicki, Teoria i praktyka rozwiązywania zadań optymalizacji, WNT, Warszawa, 2006.

ENGLISH 1

Course code: 09.0-WK-MAT-SD-JA1
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: mgr Grażyna Czarkowska
Name of lecturer: mgr Grażyna Czarkowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Laboratory	30	2	II	Grade	

COURSE AIM:

The course aims to enable students to improve speaking, reading and writing skills, as well as listening comprehension in English. It will help students to develop their ability to apply language functions to effective communication in everyday life. The course also aims to further develop students' ability to use the language of mathematics in order to discuss mathematical problems and read, with understanding, specialist texts. It also encourages students to master their skills of expressing ideas using complex language structures, e.g. Passive Voice, and grammar tenses to describe past activities. It provides an opportunity to revise the rules and master the skills of giving a presentation in English.

ENTRY REQUIREMENTS:

B1+/B2 of the Common European Framework of Reference for Languages specified by the Council of Europe.

COURSE CONTENTS:

During the course students will learn to or improve their ability to:

- describe past events using different grammar tenses (4 hours)
- understand and use Passive Voice (4 hours)
- exchange information concerning mathematical problems (2 hours)
- give definitions of integers, natural, rational, irrational, real and complex numbers (2 hours)
- read numbers and mathematical symbols (2 hours)
- use the symbols to read mathematical expressions (2 hours)
- use the language of mathematics in speakig and writing (4 hours)
- better understand specialist texts (4 hours)
- prepare and deliver a presentation on a topic concerning mathematics (2 hours)
- discuss mathematical problems in class, give arguments for and against (2 hours)
- form questions to get information concerning mathematical problems, as well as give answers to such questions (2 hours)

TEACHING METHODS:

The course focuses on communication activities in functional and situational context. It encourages students to speak with fluency and develop the four skills of reading, writing, listening and speaking by means of group and pair work, discussion, presentation, oral and written exercises.

LEARNING OUTCOMES:

Depening language skills and competence on level B2 of the Common European Framework of Reference for Languages.

Upon successful completion of the course, the students:

- are able to describe and compare past events using different grammar tenses
- understand and form Pasive Voice sentences
- can form questions about mathematical problems – number theory
- exchange information concerning mathematical problems
- understand specialist texts
- are able to write and read numbers and mathematical operations
- know how to prepare and deliver a presentation on a topic concerning mathematics
- know and use in speech the language of mathematics
- understand the need for lifelong education
- can cooperate with members of a group, exchange information, and discuss problems

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Classes – grade: a condition for receiving a credit are positive marks for tests, participating in class discussions, dialogues, delivering a presentation in English, getting information on different topics.

STUDENT WORKLOAD:

Contact time:

- classes – 30 hours
- consultation – 5 hours

Private study – 25 hours, students systematically prepare for the exam.

RECOMMENDED READING:

1. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Student's Book,* Oxford University Press 2007
2. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Workbook,* Oxford University Press 2007
3. J. Pasternak-Winiarska, *English in Mathematics*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2006

OPTIONAL READING:

1. *FCE Use of English* by V. Evans
2. L. Szkutnik, *Materiały do czytania – Mathematics, Physics, Chemistry,* Wydawnictwa Szkolne i Pedagogiczne
3. Internet articles
4. R. Murphy *English Grammar in Use*.

ENGLISH 2

Course code: 09.0-WK-MAT-SD-JA1
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: mgr Grażyna Czarkowska
Name of lecturer: mgr Grażyna Czarkowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Laboratory	30	2	II	Exam	

COURSE AIM:

The course aims to enable students to improve speaking, reading and writing skills, as well as listening comprehension in English. It will help students to develop their ability to apply language functions to effective communication in everyday life. The course also aims to further develop students' ability to use the language of mathematics in order to discuss mathematical problems and read, with understanding, specialist texts. It also encourages students to master their skills of expressing ideas using complex language structures, e.g. Passive Voice, and grammar tenses to describe present, past and future activities. It provides an opportunity to revise the rules and master the skills of giving a presentation in English.

ENTRY REQUIREMENTS:

B1+/B2 of the Common European Framework of Reference for Languages specified by the Council of Europe.

COURSE CONTENTS:

During the course students will learn to or improve their ability to:

- describe present, past and future events using different grammar tenses (4 hours)
- understand and use Passive Voice, especially in mathematical texts(4 hours)
- exchange information concerning mathematical problems – plane geometry, set theory (2 hours)
- give definitions of sets (2 hours)
- read mathematical symbols from set theory (2 hours)
- understand vocabulary used to describe angles and geometric figures (2 hours)
- use properly the language of mathematics in speaking and writing (4 hours)
- better understand specialist texts (4 hours)
- prepare and deliver a presentation on a topic concerning mathematics (2 hours)
- discuss mathematical problems in class, give arguments for and against (2 hours)
- understand expressions used in mathematical proofs and abstracts (2 hours)

TEACHING METHODS:

The course focuses on communication activities in functional and situational context. It encourages students to speak with fluency and develop the four skills of reading, writing, listening and speaking by means of group and pair work, discussion, presentation, listening, oral and written exercises.

LEARNING OUTCOMES:

Depening language skills and competence on level B2 of the Common European Framework of Reference for Languages.

Upon successful completion of the course, the students:

- are able to describe and compare past,present and future events using different grammar tenses
- understand and form Pasive Voice sentences, especially in mathematical context
- can form questions about mathematical problems –set theory, plane geometry
- exchange information concerning discussed mathematical problems
- understand specialist texts
- know definitions of angles
- know how to prepare and deliver a presentation on a topic concerning mathematics
- know and use in speech the language of mathematics
- know rules for writing abstracts
- know and understand expressions used in mathematical proofs
- understand the need for lifelong education
- can cooperate with members of a group, exchange information, and discuss problems

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Classes – grade: a condition for receiving a credit are positive marks for tests, participating in class discussions, dialogues, delivering a presentation in English, getting information on different topics.

STUDENT WORKLOAD:

Contact time:

- classes – 30 hours
- consultation – 5 hours

Private study – 25 hours, students systematically prepare for the examination.

RECOMMENDED READING:

1. J. Pasternak-Winiarska, *English in Mathematics*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2006

OPTIONAL READING:

1. *FCE Use of English* by V. Evans
2. Internet articles
3. R. Murphy *English Grammar in Use*.

OPERATIONS RESEARCH

Course code: 11.1-WK-MAT-SD-BO
Type of course: optional
Language of instruction: English
Director of studies: prof. Andrzej Cegielski
Name of lecturer: prof. Andrzej Cegielski, prof. Zbigniew Świtalski, dr. Robert Dylewski,

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	15	1	III	Examination	
Laboratory	30	2		Written test	

COURSE AIM:

The lecture should give a knowledge on mathematical foundations of operations research, in particular on foundations of discrete programming and network problems. Furthermore, basic methods for discrete problems will be presented.

ENTRY REQUIREMENTS:

Discrete mathematics, linear algebra, mathematical programming

COURSE CONTENTS:

1. Methods of operations research
2. Construction of optimization's models, examples
3. Discrete optimization and its applications
4. Optimization problems in project scheduling. CPM-COST method
5. Models and algorithms of job sequencing
6. Multicriterial programming
7. Matching models

TEACHING METHODS:

Traditional lecture, laboratory with application of appropriate software

LEARNING OUTCOMES:

Student

- Knows various models of economic problems (K_W06++)
- Is able to apply branch and bound methods to various discrete optimization problems (K_U02++, K_U03++)
- Knows and is able to apply selected methods in project scheduling and job sequencing (K_W12++, K_U02++, K_U03++)
- Understands the necessity of an application of quantity methods in practical problems (K_K04++)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Checking the activity of the student
2. Written tests
3. Checking the ability of application of an appropriate software
4. Written examination

The final grade consists of the classes grade (30%), the lab's grade (30%) and the examination's grade (40%)

STUDENT WORKLOAD:

Contact hours

- Participation in lectures: 15h
- Participation in laboratory: 30h
- Consultations: 10h (lecture), 15h (laboratory)
- Examination: 2h

Independent work

- Preparation for lectures: 8h
- Preparation for laboratory: 35h
- Preparation for examination: 35 h

Total: 150 h (6 ECTS)

RECOMMENDED READING:

1. Z. Galas, I. Nykowski, Z. Sółkiewski, Programowanie wielokryterialne, PWE, Warszawa, 1987.
2. W. Grabowski, Programowanie matematyczne, PWE, Warszawa, 1982.
3. M. Gruszczyński, T. Kuszewski, M. Podgórska, Ekonometria i badania operacyjne, PWN, Warszawa.
4. B. Guzik (red.), Ekonometria i badania operacyjne, zagadnienia podstawowe, wyd. III, Wydawnictwo AE w Poznaniu, Poznań, 2000.
5. Z. Jędrzejczyk, K. Kukuła, J. Skrzypek, A. Walkosz, Badania operacyjne w przykładach i zadaniach, wyd. IV, PWN, Warszawa, 2002.
6. W. Sikora (red.), Badania operacyjne, PWE, Warszawa, 2008.
7. T. Szapiro (red.), Decyzje menedżerskie z Excelem, PWE, Warszawa, 2000.
8. T. Trzaskalik, Wprowadzenie do badań operacyjnych z komputerem, PWE, Warszawa, 2003.
9. R. J. Vanderbei, Linear Programming, Foundations and Extensions, Kluwer, Boston, 1997.

PARTIAL DIFFERENTIAL EQUATIONS

Course code: 11.1-WK-MAT-SD-RRC
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Tomasz Małolepszy
Name of lecturer: dr Tomasz Małolepszy

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					10
Lecture	30	2	III	Exam	
Class	30	2		Grade	
Laboratory	30	2		Grade	

COURSE AIM:

The main aim of this course is to acquire by students skills to solve the initial-boundary value problems (IBVP) for linear PDE of first and second orders by the means of the method of the characteristics, the method of the separation of variables and Fourier transform. During that course students also will learn the basics of the theory of Sobolev spaces and so-called weak formulation of IBVP for some PDE.

ENTRY REQUIREMENTS:

Mathematical Analysis 1 and 2, Functional Analysis, Linear Algebra 1 and 2

COURSE CONTENTS:

1. Basic definitions - linear, semilinear and nonlinear equations, Cauchy problems, the types of boundary problems, characteristic surfaces.
2. Equations of the first order. The method of the characteristics. Cauchy-Kowalewski theorem.
3. Equations of the second order. Classification of the second order equations.
   a. Elliptic equations - basic properties of the harmonic functions, the fundamental solution to Laplace's and Poisson's equations, the maximum principles, Green's function for elliptic equation.
   b. Parabolic equation - the fundamental solution of the Cauchy problem for the heat equation, the maximum principles, the method of the separation of variables.
   c. Hyperbolic equations - D'Alembert formula, formulas for the solutions of the wave equation in higher dimensions, Duhamel's principle.
4. Fourier transform and its application in the theory of partial differential equations.
5. Elements of the theory of Sobolev spaces.
   a. Weak derivatives.
   b. Sobolev spaces.
   c. Approximation of the elements of the Sobolev spaces by smooth functions.
   d. Trace of the function.
   e. Sobolev-type inequalities.
6. Weak solutions of the second order equations - the methods of Ritz and Galerkin.

TEACHING METHODS:

Traditional lectures; classes with the lists of exercises to solve by students; computer lab.

LEARNING OUTCOMES:

Student is able:

1. to solve I order quasilinear PDEs with the use of characteristic method and Lagrange method, (K_W10++, K_U06++)
2. to find the canonical form of II order semilinear PDEs, (K_W10+, K_U06+)
3. to use a method of separation of variables to solve initial-boundary value problems for II order linear PDEs, (K_W10++, K_U05+, K_U06++, K_U16+)
4. to define weak derivatives and Sobolev spaces, (K_U06++, K_U09+, K_K06+)
5. to use basic numerical methods (finite difference method, finite element method) to find solutions of some PDEs. (K_W11++, K_K01+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Class and Laboratory: learning outcomes will be verified through two tests consisted of exercises of different degree of difficulty. A grade determined by the sum of points from these two tests is a basis of assessment.

Lecture: final exam. A grade determined by the sum of points from that exam is a basis of assessment.

A grade from the course is consisted of the grade from laboratory (25%), the grade from classes (25%) and the grade from the final exam (50%). To take a final exam, students must receive a positive grade from classes. To attain a pass in the course students are required to pass the final exam.

STUDENT WORKLOAD:

Contact hours

Lectures - 30 hours.

Classes - 30 hours.

Laboratories - 30 hours.

Lectures' consultation hours - 5 hours.

Classes' consultation hours - 5 hours.

Laboratories' consultation hours - 5 hours.

Total - 105 hours (4 ECTS).

Individual work

Preparation to lectures - 30 hours.

Preparation to classes - 40 hours.

Preparation to laboratories - 40 hours.

Preparation to the final exam - 30 hours.

Total - 140 hours (6 ECTS).

Total time needed for this course: 245 hours (10 ECTS).

RECOMMENDED READING:

1. *Warsztaty z Równań Różniczkowych Cząstkowych*, pod red. naukową prof. dr. hab. P. Bilera, Torun, 2003.
2. Evans, L., *Partial differential equations*, AMS, 1998.
3. Marcinkowska, H., *Dystrybucje, przestrzenie Sobolewa, równania różniczkowe*, PWN, 1993.
4. Walter A. Strauss, *Partial differential equations: an introduction*, Wiley, New York 1992.

OPTIONAL READING:

1. Strzelecki, P., *Krótkie wprowadzenie do równań różniczkowych cząstkowych*, Wydawnictwa Uniwersytetu Warszawskiego, 2006.

REAL AND COMPLEX ANALYSIS

Course code: 11.1-WK-MAT-SD-ARZ
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: prof. dr hab. Janusz Matkowski
Name of lecturer: prof. dr hab. Witold Jarczyk
prof. dr hab. Janusz Matkowski
dr Justyna Jarczyk

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	I	Exam	
Class	30	2		Grade	

COURSE AIM:

The aim is to improve the acquaitance of a student of deeper facts in real analysis and give him opportunity to gain the standard knowledge in the theory of complex functions in single variable.

ENTRY REQUIREMENTS:

Average education in the basic notions and results in real analysis.

COURSE CONTENTS:

Lecture

I. MEASURE THEORY
1. Theorems of Jegorov, Lusin (4 h.).
2. Theorems of Fubini and Radon-Nikodym (4 h.).

II. THEORY OF COMPLEX FUNCTIONS
1. Complex derivative, Cauchy-Riemann equations, analytic (holomorphic) function (4 h.).
2. Curve integral of a complex function, Cauchy integral theorem, Cauchy's integral formula (4 h.).
3. Expansion of an analytic function in power series, entire functions, theorem of Liouville, maximum principle, Schwarz lemma (5 h.).
4. 4. Laurent series, singular points and their classification, residuum (5 h.).
5. 5. Theorem of residues and their applications, meromorphic functions (4 h.).

Exercises

I. MEASURE THEORY
1. Thorems of Jegorov, Lusin (3 h.)
2. Theorems of Fubini and Radon-Nikodym (3 h.)

II. THEORY OF COMPLEX FUNCTIONS
1. Complex derivative, Cauchy-Riemann equations, analytic (holomorphic) function (4 h.).
2. Curve integral of a complex function, Cauchy integral theorem, Cauchy's integral formula (6 h.).
3. Expansion of an analytic function in power series, entire functions, theorem of Liouville, maximum principle, Schwarz lemma (5 h.).
4. Laurent series, singular points and their classification, residuum (5 h.).
5. Theorem of residues and their applications, meromorphic functions (4 h.).

TEACHING METHODS:

Conventional lecture; problem lecture
Auditorium exercises – solving standard problems enlightening the significance of the theory, exercises on applications, solving problems.

LEARNING OUTCOMES:

1. Student is able to formulate the basic results in measure theory (K W01+,K W03+, K W07).
2. Student is able to apply the Radon-Nikodym theorem in probability theory (K U07+).
3. Student defines the derivative of a complex function, is able to present its interpretation derivative, and distinguishes the differences between the real and complex analysis (K W04+, K W07+).
4. Student knows the fundamental theorems of Cauchy, their proofs, and is able to apply them in calculating the integrals (K W02+, K W03+, K U05+).
5. Student is able to expand an analytic function in annular neighborhood of a point in Laurent series and distinguish its singularities of (K U05+).
6. Student knows idea of residuum of function and is able to apply them in calculating the integrals (K U05+).
7. Student is self-sufficient in finding the bibliographical information (K U19+).

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Examination of the students' preparation and their activity during exercises.
2. Tests, of different level of difficulty, permitting to verify the level of student commanding of the particular effects of education.
3. Exam (written and oral) checks the understanding of the basic notions, knowledge of the important examples and the proofs of some chosen theorems.
   Passing the exam: the weighted mean of notes of exercises (40%) and the exam (60%).
   A positive note of the exercises is the necessary condition to be admitted to the exam. A positive note of the exam attests the subject.

STUDENT WORKLOAD:

Contact hours
Lectures – 30 h.
Exercises – 30 h.
Office ours – 5 h. for lectures + 10 h. for exercises.
Jointly : 75 h. (3 ECTS)
Self-educational work
Preparation for the lecture – 25 h.
Preparation for the exercise – 30 h.
Preparation for the exam – 30 h.
Preparation for the tests – 25 h.
Jointly : 110 h. (4 ECTS)
Entire subject jointly: 185 h. (7 ECTS).

RECOMMENDED READING:

1. Franciszek Leja, Funkcje zespolone, Biblioteka Matematyczna, PWN, 1973; Rozdziały VII-IX.
2. Walter Rudin, Real and Complex Analysis, Third Edition, Mc Graw - Hill Company, 1987.
3. B.W. Szabat, Wstęp do analizy zespolonej, Wydawnictwo PWN, Warszawa 1974.

OPTIONAL READING:

1. Roman Sikorski, Funkcje rzeczywiste I, Państwowe Wydawnictwo Naukowe, Warszawa 1957.
2. W. Kołodziej, Analiza matematyczna, Państwowe Wydawnictwo Naukowe, Warszawa

STOCHASTIC PROCESSES 1

Course code:	11.1-WK-MAT-SD-PS1
Type of course:	optional
Language of instruction:	English/Polish
Director of studies:	prof. dr. hab. Andrzej Nowak
Name of lecturer:	prof. dr. hab. Andrzej Nowak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	II or IV	Exam	
Class	30	2		Grade	

COURSE AIM:

Knowledge of basic terminology and theory of stochastic processes and its applications.

ENTRY REQUIREMENTS:

Mathematical analysis 1 i 2, linear algebra, probability theory

COURSE CONTENTS:

Lectures

I. Homogenous Markov chains:

1. Transition probability matrix. Chapman - Kołmogorov equation (2 hrs)
2. Classification of states. (2 hrs)
3. Random walk. Player's ruin problem (2 hrs)
4. Stationarity and ergodicity of Markov chain. (2 hrs)

II. Poisson Process:

1. Construction of Poisson process. (2 hrs)
2. Compound and conditional Poisson process. (2 hrs)
3. Applications of processes of this kind . (4 hrs)

III. Continuous-time Markov chains:

1. Birth-death process. (2 hrs)
2. Species extinction problems. (2 hrs)
3. Application of Poisson process. (2 hrs)

IV. General properties of stochastic processes:

1. Existence of process with given distributions. (2 hrs)
2. Stochastic equivalence and separability of processes. (2 hrs)

V. Wiener Process:

1. Properties of trajectory. (2 hrs)
2. Law of the iterated logarithm. (2 hrs)

Class

I. Homogenous Markov chains:

1. Examples of transition probabilisty matrces. (2 hrs)
2. Classification of states. (2 hrs)
3. Random walks. Problems (3 hrs)
4. Stationarity and ergodicity of Markov chains. Examples. (3 hrs)

II. Poisson process:

1. Problems on properties of Poisson processes. (2 hrs)
2. Compound and conditional Poisson process. Examples. (3 hrs)
3. Applications of processes of this kind. (3 hrs)

III. Continuous-time Markov chains:

1. Birth-death process. (2 hrs)
2. Applications and examples. (3 hrs)

IV. General properties of stochastic processes

1. Existence of process with given distributions. (1 hr)
2. Stochastic equivalence and separability of processes. (1 hr)

V. Wiener processes:

1. Properties of trajectory. Correlation functions. (1 hr)

VI. Colloquium: (4 hrs).

TEACHING METHODS:

Lectures and classes

LEARNING OUTCOMES:

Student

1. understands significance of stochastic processes in mathematics, and other branches of sciences and economic models (K_W04, K_W07),
2. knows fundamental theorems on Markov chains and its applications (K_W04),
3. knows construction of Poisson process, properties of its trajectories (K_W04),
4. can interpret and explain continuous time Markov processes and apply to practical problems (K_U11, K_U16),
5. knows basic theorems of general stochastic processes (K_W04),
6. knows and understand basic properties of Wiener process (K_W04, K_U04),

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Evaluation of individual exercises, final exam and grade.

STUDENT WORKLOAD:

Contact hours

Lecture – 30 hrs

Class – 30 hrs

Office hours – 15 hrs (5 hrs for lecture and 10 for class)

Total 75 hrs (3 ECTS)

Individual work

Preparing to lecture – 25 hrs

Preparing to class – 35 hrs

Preparing to exam – 40 hrs

Total: 100 hrs (4 ECTS)

Total hours and points per course: 175 hrs (7 ECTS)

RECOMMENDED READING:

1. Feller, W. Wstęp do rachunku prawdopodobieństwa, T.1, 2. PWN, Warszawa, 2009.
2. Iwanik, A., Misiewicz, J. Wykłady z procesów stochastycznych z zadaniami. Część I. Script, Warszawa 2010.

OPTIONAL READING:

1. Billingsley, P. Prawdopodobieństwo i miara. PWN, Warszawa, 2009.

STOCHASTIC PROCESSES 2

Course code: 11.1-WK-Mat-SD-PS2
Type of course: optional
Language of instruction: Polish/English
Director of studies: prof. dr hab. Jerzy Motyl
Name of lecturer: prof. dr hab. Jerzy Motyl

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	III	Exam	
Class	30	2		Grade	

COURSE AIM:

After the course of "stochastic processes 2" students should be able to solve themselves practical and theoretical problems on the topic.

ENTRY REQUIREMENTS:

Probability theory, mathematical analysis, functional analysis

COURSE CONTENTS:

Lecture:

Introduction (5 h.)

1. Stochastic processes in practical problems
2. Elements of stochastic analysis, stochastic processes, definition and properties, Kołmogorov's theorem
3. Wiener process: existence and properties

Stochastic square-mean analysis (13 h.):

1. Hilbert process and different types of its convergences
2. Square-mean continuity and differentiability of Hilbert processes
3. Square-mean integrals of Riemann and Lebesgue type
4. Square-mean integrability
5. Variation of stochastic processes, existence of Riemann-Stieltjes and Lebesgue-Stieltjes trajectory integrals

Stochastic Itô integral (7 h.):

1. Wiener filtration and adapted processes
2. Simple processes and their Wiener integrals
3. Convergence of simple processes to process from $M^{[a,b]}$ and convergence of their integrals in $L^2(\Omega)$
4. Stochastic Itô integral and its properties
5. Itô formula and its applications
6. Stochastic Itô differential equations

Class

Properties of random variables
Properties of stochastic processes
Convergence of stochastic processes
continuity and differentiability of Hilbert processes
Stochastic differentials of different processes
Applications of Itô formula
Solving of stochastic Itô differential equations

TEACHING METHODS:

Conventional lecture; problem lecture
Auditorium exercises – solving standard problems enlightening the significance of the theory, exercises on applications, solving problems.

LEARNING OUTCOMES:

1. K_W04 has in-depth knowledge in the chosen field of theoretical mathematics or applied
2. K_U03 has the ability to validate evidence of formal building of proofs
3. _U09 uses the language and methods of functional analysis in mathematical analysis and its applications, in particular property uses the classic Banach spaces and Hilbert
4. K_U14 in the selected field can carry out evidence which, if necessary, also the tools from other departments of mathematics
5. K_K01 knows the limitations of his own knowledge and understands the need for further education
6. K_K04 is able to formulate opinions on the basic issues of mathematical proofs

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Final exam and grade

STUDENT WORKLOAD:

Lectures - 30 h
Classes - 30 h
Tutoring – 15 h (Lectures - 5 h; Classes - 10 h)
Total: 75 h (3 ECTS)
Individual students` work
Preparing to lectures - 30 h
Preparing to classes - 30 h
Preparing to the exam - 40 h
Total: 100 h (4 ECTS)
Total hours and points per course 175 h (7 ECTS)

RECOMMENDED READING:

1. R. Lipcer, A. Sziriajew, Statystyka procesów stochastycznych, PWN 1981.
2. K. Sobczyk, Stochastyczne równania różniczkowe, WNT 1996.
3. M. Fisz, Rachunek prawdopodobieństwa i statystyka matematyczna, PWN 1958.

OPTIONAL READING:

1. E. Parzen, Stochastic processes, Holden-Day Inc. 1962.
2. C.W. Gardiner, Handbook of stochastic methods for Physics, Chemistry and the Natural Sciences, Springer-Verlag 1985.

TOPICS IN DISCRETE MATHEMATICS

Course code: 11.1-WK-IiE-SD-WZMD
Type of course: optional
Language of instruction: English
Director of studies: dr Elżbieta Sidorowicz
Name of lecturer: dr Elżbieta Sidorowicz

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	II or IV	Exam	
Class	30	2		Grade	

COURSE AIM:

The course introduce the advanced notions and ideas of discrete mathematics in theoretical and algorithmic aspects

ENTRY REQUIREMENTS:

Discrete Mathematics 1

COURSE CONTENTS:

1. Hypergraphs, basic properties and the representation.
2. Characterization of classes of hypergraphs and their recognition algorithms.
3. Colourings of hypergraphs and the complexity of this problem.
4. The transversal and covering of hypergraphs.
5. The intersection graph and the middle graph. The algorithmic properties of these graphs and their applications.
6. New directions in hypergraph theory.

TEACHING METHODS:

Lecture: the traditional oral essay, the participatory lecture.

Class: solving selected problems, applying the theory for solving problems.

LEARNING OUTCOMES:

1. Student knows the basic definitions, properties and theorem related with graphs and hypergraphs. (K_W01,K_W04)
2. Student can applies theorems to determine graphs invariants. (K_U01, K_U13)
3. Student is able to prepare and present a talk on the particular topic. (K_U02, K_U18)
4. Student knows its own limitation of knowledge and understands the need for further learning. (K_K01) Student understands the need for lifelong education.

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Verifying the level of preparation of students and their activities during the classes.
2. Two written tests.
3. The talk.
4. The written and oral exam.

Assessment criteria:

the mean of the assessment and evaluation of lectures and exams (written and oral)

The necessary condition for taking the exam is positive assessment of two tests (with tasks of different difficulty which help to assess whether students have achieved effects of the course in a minimum degree), positive assessment of the talk and active participation in the classes.

The necessary condition for passing the course is the positive assessment of the exam.

STUDENT WORKLOAD:

lecture – 30 hours
class – 30 hours
consultation – 8 hours
exam – 3 hours
preparing to class – 45 hours
preparing to tests – 15 hours
preparing to lectures – 9 hours
preparing to the exam – 20 hours
preparing to the talk – 15 hours
Sum for the course: 175 hours (7 ECTS)

RECOMMENDED READING:

1. C. Berge, Graphs and Hypergraphs, North-Holland, Amsterdam 1973.
2. Branstadt, V.B Le, J.P. Spinarad, Graph Classes - A survey.

OPTIONAL READING:

1. Recent papers on these topics.

TOPOLOGY

Course code: 11. 1-WK-Mat_SD-T
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Andrzej Kisielewicz
Name of lecturer: dr Andrzej Kisielewicz

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	I	Exam	
Class	30	2		Grade	

COURSE AIM:

The basic notions of algebraic and geometric topology.

ENTRY REQUIREMENTS:

General topology, group theory.

COURSE CONTENTS:

Lecture

The Fundamental group
1. Homotopy (2 h)
2. Retractions (1 h)
3. Construction of the fundamental group (3 h)
4. The Fundamental group of the Cartesian product (1 h)
5. Symplices and symplicial complexes (2 h)
6. Calculating of the fundamental groups (2 h)
7. The fundamental group of the circle, the torus, the sphere, the projective plane (2 h)

The Jordan theorem (proof), the Schoenfliesa theorem (3 h)
Topology in art – Alexander's sphere, Wady's leaks, art of M.C. Escher (2 h)
Classification Theorem for Surfaces
1. Surfaces (1 h)
2. Polytopes (1 h)
3. Triangulation of surfaces (1 h)
4. The proof of Classification Theorem for Surfaces (2 h)

The Borsuka-Ulama theorem
1. The various forms of the Borsuk-Ulam theorem (2 h)
2. The Tucker lemma and the proof of the Borsuk-Ulam theorem (2 h)
3. Applications of the Borsuk-Ulam theorem (2 h)
4. The Brouwer fixed-point theorem (2 h)

Degree of mappings. (2 h)

Class
Topologies
1. Basic exercises on topologies (1 h)
2. Examples of topologies (1 h)
Homotopy
1. Exercises on homotopy and equivalence relations (2 h)
2. Exercises dealing with the construction of fundamental group (3 h)
3. Exercises on retractions (1 h)
4. Exercises on the fundamental group (3 h)
Classification Theorem for Surfaces
1. -Exercises on classification of surfaces (2 h)
2. Exercises on triangulations of surfaces (1 h)
The Borsuka-Ulama theorem
1. Proofs of various versions of The Borsuka-Ulama theorem (4 h)
2. Exercises which use The Borsuka-Ulama theorem (2 h)
3. Proof of the Sperner lemma (2 h)
Presentations and class tests (6 h)

TEACHING METHODS:

Lectures and disccusions

LEARNING OUTCOMES:

1. Students understand the importance of the fundamental group (K_U04)
2. Students are able to decide, in simple situations, whether two object are homeomorphic (K_U08)
3. Students are able, in simple situations, to compute the fundamental group (K_0U8)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Exams and talks

STUDENT WORKLOAD:

1. Lecture-30 hours
2. Class-30 hours
3. Consultations-10 hours

Individual work

1. Lecture preparation -40 hours
2. Class preparation-40 hours
3. Exams preparation-30 hours

RECOMMENDED READING:

1. Roman Duda, Wprowadzenie do topologii I, II, PWN, 1986.
2. Jiri Matousek, Using the Borsuk-Ulam theorem, Springer, 2003

OPTIONAL READING:

1. Jerzy Mioduszewski, Wykład z topologii, Wydawnictwo Uniwersytetu Śląskiego, 1994
2. Allen Hatcher, Algebraic Topology, www.math.cornell.edu/~hatcher/