M A T H E M A T I C S

STUDIES OF FIRST DEGREE

COURSE DIRECTORY

Since 2013/2014

ALGORITHMS AND DATA STRUCTURED

Course code: 11.3-WK-MAT-SP-ASD
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Florian Fabiś
Name of lecturer: dr Florian Fabiś,
mgr Katarzyna Jesse-Józefczyk

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					5
Lecture	30	2	V	Exam	
Laboratory	30	2		Grade	

COURSE AIM:

The knowledge and skills in basics of analysis of algorithms. The knowledge of and ability to implement sorting and selection algorithms, searching algorithms and elementary graph algorithms. The knowledge of NP-completeness problem and its practical aspects.

ENTRY REQUIREMENTS:

Gaining of basic competences in analysis and linear algebra. Advanced computer operating skills. Skills in computer structured programming.

COURSE CONTENTS:

Lecture

1. Algorithms. Computational complexity of algorithms. Correctness of algorithms. Asymptotics. (4 h)
2. Techniques of constructing effective algorithms: divide and conquer , greedy methods, dynamic programming. (2 h)
3. Algorithms of sorting and searching. (4 h)
4. Data structures for dictionaries: characteristic vector, binary search trees, hashing. External searching - B-trees. The union problem for disjoint sets and its applications. (6 h)
5. Graph algorithms: computer representations of graphs, graph searching, minimum spanning trees, shortest paths in graphs, flows in networks. (4 h)
6. Text algorithms: pattern matching, suffix trees. (4 h)
7. Computational geometry: point localization, convex hull, sweeping. (4 h)
8. NP-completeness: the classes P, NP and NP-complete.

Laboratory

1. Determination of the computational complexity of algorithms. (4 h)
2. Testing of the correctness of algorithms. (4 h)
3. Algorithms of sorting and searching. (4 h)
4. Data structures for dictionaries. (6 h)
5. Graph algorithms. (6 h)
6. Text and computational geometry algorithms. (6 h)

TEACHING METHODS:

Lecture: problem lecture.

Laboratory: laboratory exercises in computer lab – implementation and testing of selected algorithms.

Each student is supposed to realize four projects during the semester. Each project will consist in implementation of the selected algorithm to solve a concrete practical task, writing a program for it, testing it and presenting a documentation in accordance with the assigned specification. On two out of the four projects the students will work in 2-3 person groups. Furthermore the students will write on classes programs implementing other algorithms.

LEARNING OUTCOMES:

Student knows and understands basic concepts and mathematical description used in the analysis of algorithms. [K_W08+++]

Student is able to design and analyze algorithm according to specification assigned [K_U26++].

Student knows the basic data structures for dictionaries and can implement them in programs. [K_W08++]

Student knows the basic sorting, searching, graph, text and computational geometry algorithms and can implement them in programs. [K_U26++, K_W08++]

Student recognizes the problems that can be solved algorithmically and can make a problem specification. [K_U25++].

Student is able to work in project team. [K_K03++]

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Lecture. Written examination verifying the education outcome in area of knowledge and skills.

Laboratory. Final grade is granted based on number of points received during studies. Points are received for written tests, active participation in classes and completed project.

Final course grade consists of laboratory classes' grade (50%) and examination grade (50%). Positive grade from laboratory classes is the necessary condition for participation in examination. The positive grade from examination is the necessary condition for course completion.

STUDENT WORKLOAD:

Contact hours

- Participation in lectures: 15*2 h = 30 h
- Participation in laboratory studies : 15*2 h = 30 h
- Consultations: = 8 h
- Participation in the exam:1*2 h = 2 h

Total: 70 h (3 ECTS)

Independent work

- Preparation for laboratory exercises: 15*1 h = 15 h
- Finishing in house exercise laboratory: 15*1 h = 15 h
- Exam preparation: 20 h

Total: 50 h (2 ECTS)

Total for the course: 120 h (5 ECTS)

RECOMMENDED READING:

1. Aho A., Hopcroft J.E., Ullman J.D., : Projektowanie i analiza algorytmów komputerowych, PWN, Warszawa 1983.
2. Banachowski L., Diks K., Rytter W., Algorytmy i struktury danych, WNT, W-wa 1996.
3. Cormen T.H., Leiserson C.E., Rivest R.L., Wprowadzenie do algorytmów, WNT, Warszawa 1997.

OPTIONAL READING:

1. Aho A., Hopcroft J.E., Ullman J.D., : The Design and Analysis of Computer Algorithms.
2. Aho A., Hopcroft J.E., Ullman J.D., : Data structures and algorithms
3. T.H. Cormen, Ch.E. Leiserson, R.L. Rivest: Introduction to Algorithms, 2001, MIT Press.
4. Knuth D. E. : The Art of Computer Programming.
5. N. Wirth: Algorithms and Data Structured, 1985.
6. Błażewicz J. : Złożoność obliczeniowa problemów kombinatorycznych, WNT, Warszawa 1988.
7. P. Wróblewski: Algorytmy, struktury danych i techniki programowania, wyd. II popr., Helion, 2001.

COMPUTER PROGRAMMING 1

Course code: 11.3-WK-MAT-SP-PK1
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Florian Fabiś
Name of lecturer: dr Florian Fabiś,
dr inż. Mariusz Hałuszczak,
mgr Katarzyna Jesse-Józefczyk

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	I	Exam	
Laboratory	30	2		Grade	

COURSE AIM:

The ability of elementary programming in Pascal with application of basic data structures, with regard to principles of structured programming. Knowledge and skills in basics of analysis of algorithms. The knowledge of basic methods of effective algorithms' constructing.

ENTRY REQUIREMENTS:

Advanced skills in computer operating. The competences in maths at secondary school level.

COURSE CONTENTS:

Lecture

1. **Computer system**. Hardware and software. Algorithm, program, compiler, block diagram. (2 h)
2. **Structured programming in Pascal.** Programming paradigms. Program structure and syntax. Data simple types: integer, real, boolean, character, string. Variables declarations. Assignment statements. Input and output standard procedures. Procedures and functions: declaring procedures and functions, local declarations, value and variable parameters, constant parametrs, untyped parameters, string parameters, array parameters, open parameters, recursion, standard procedures and functions. Conditional and repetitive statements. Arrays. Strings (18 h)
3. **Algorithmics.** Computational complexity. Correctness of algorithms**.** Asymptotics. (5 h)
4. **Techniques of constructing effective algorithms.** Divide and conquer , greedy methods, dynamic programming. **The basic algorithms of sorting.** (5 h)

Laboratory

1. Block diagrams. (2 h)
2. Data simple types. Variables declarations. Assignment statements. Input and output standard procedures. (2 h)
3. Design, code, and debug simple programs with assignment statements, conditional statements and input-output standard procedures. (2 h)
4. Design, code, and debug programs with application of procedures and functions. (2 h)

5. Design, code, and debug programs with application of repetitive statements. (6 h)
6. Processing of arrays. (10 h)
7. Processing of strings. (4 h)
8. Run self written application, including all Pascal elements learned during the classes with documentation according to assigned specification. (2 h)

TEACHING METHODS:

Lecture: problem lecture.

Laboratory: laboratory exercises in computer lab – writing and running programs on assigned leading themes, analysis of these programs and analysis of algorithms applied. Students will work on some programs in groups consisting of 2-3 persons.

Besides, each student is required to present on last classes a self written application, including all Pascal elements learned during the classes, with documentation according to assigned specification.

LEARNING OUTCOMES:

After course completion student is able to design and analyze algorithm based on specification. [K_U26+].

Student is able to interpret and analyze exemplary programs in Pascal. [++]

Student can design, write and debug simple programs using basic elements of Pascal. [K_U27++].

Student is able to prepare program documentation according to specification assigned. [K_U26+]

Student has basic knowledge about computers arithmetic, paradigms of programing, algorithms and computational complexity. [K_W08+]

Student is able to work in group. [K_K03+]

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Lecture. Written examination verifying the education outcome in area of knowledge and skills.

Laboratory. Final grade is granted based on number of points received during studies. Points are received for written tests, active participation in classes and on last classes presented application.

Final course grade consists of laboratory classes' grade (60%) and examination grade (40%). Positive grade from laboratory classes is the necessary condition for participation in examination. The positive grade from examination is the necessary condition for course completion.

STUDENT WORKLOAD:

Contact hours

- Participation in lectures: 15*2 h = 30 h
- Participation in laboratory studies : 15*2 h = 30 h
- Consultations: = 8 h
- Participation in the exam:1*2 h = 2 h

Total: 70 h (3 ECTS)

Independent work

- Preparation for laboratory exercises: 15*2 h = 30 h
- Finishing in house exercise laboratory: 15*1 h +5 h = 20 h
- Exam preparation: 20 h

Total: 70 h (3 ECTS)

Total for the course: 140 h (6 ECTS)

RECOMMENDED READING:

1. Banachowski L., Diks K., Rytter W. : Algorytmy i struktury danych, WNT, W-wa
2. Koleśnik K.: Wstęp do programowania z przykładami w Turbo Pascalu, Helion, 1999.
3. Sielicki A. (pod red.) : Laboratorium programowania w języku Pascal, Pol. Wr., Wrocław 1996.

OPTIONAL READING:

1. Cormen T.H,. Leiserson Ch.E, Rivest R.L. : Introduction to Algorithms, MIT Press, 2001.
2. Wirth N.: Algorithms and Data Structured, 1985.
3. Kwasowiec W.: Wprowadzenie do Object Pascal i Delphi, MIKOM, 2002.
4. Szmit M.: Delphi, Helion, 2006.

COMPUTER PROGRAMMING 2

Course code: 11.3-WK-MAT-SP-PK2

Type of course: compulsory

Language of instruction: English/Polish

Director of studies: dr Florian Fabiś

Name of lecturer: dr Florian Fabiś,
dr inż. Mariusz Hałuszczak,
mgr Katarzyna Jesse-Józefczyk

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	II	Exam	
Laboratory	30	2		Grade	

COURSE AIM:

The ability of advanced programming in Pascal: processing of arrays and records, saving to and reading from file, dynamic data structures. The ability to implement sorting, selection and searching algorithms in programs. The basic knowledge of NP-completeness problem (classes P, NP, NP – complete).

ENTRY REQUIREMENTS:

Computer programming 1

COURSE CONTENTS:

Lecture

1. **Structured programming in Pascal.** Structured types: arrays, sets, records. Unit structure and syntax.File types. Pointers and pointer types. Dynamic data structures: stacks, queues, lists. (24 h)
2. **The basic algorithms of searching.** Linear search, binary search, interpolation search. (2 h)
3. **NP-completness**. Optimization and decision problems. Models of computation: deterministic Turing machine, non-deterministic Turing machine, RAM machine. The classes P, NP and NP-complete.(4 h)

Laboratory

1. Writing and running of programs containing the algorithms for sorting and selection. Analysis of the computational complexity of these algorithms. (4 h)
2. Units. (2 h)
3. Processing arrays of records. (5 h)
4. Processing files. (6 h)
5. Dynamic data structures: dynamic arrays, stacks, queues, lists. (7 h)
6. Writing of program be simple database. (6 h)

TEACHING METHODS:

Lecture: problem lecture.

Laboratory: laboratory exercises in computer lab – writing and running programs on assigned leading themes, analysis of these programs and analysis of algorithms applied. Students will work on some programs in groups consisting of 2-3 persons.

Besides, each student is required to present on last classes a self written application, including all Pascal elements learned during the classes, with documentation according to assigned specification.

LEARNING OUTCOMES:

After course completion student is able to compile, run and test advanced program written in Pascal. [K_U27++]

Student is able to carry out an analysis of the computational complexity simpler algorithms. [++]

Student recognizes the problems that can be solved algorithmically and can make a problem specification. [K_U25++]

Student knows the basic algorithms for sorting and searching, and can implement them in programs written in Pascal. [++]

Student has a basic knowledge of NP-complete problems [K_W08+]

Student can use ready-made libraries without disturbing someone else's intellectual property. [K_K04+]

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Lecture. Written examination verifying the education outcome in area of knowledge and skills.

Laboratory. Final grade is granted based on number of points received during studies. Points are received for written tests, active participation in classes and on last classes presented application.

Final course grade consists of laboratory classes' grade (60%) and examination grade (40%). Positive grade from laboratory classes is the necessary condition for participation in examination. The positive grade from examination is the necessary condition for course completion.

STUDENT WORKLOAD:

Contact hours

- Participation in lectures: 15*2 h = 30 h
- Participation in laboratory studies: 15*2 h = 30 h
- Consultations: = 8 h
- Participation in the exam:1*2h = 2 h

Total: 70 h (3 ECTS)

Independent work

- Preparation for laboratory exercises: 15*2 h = 30 h
- Finishing in house exercise laboratory: 15 *1h = 15 h
- Work on the final application : 10 h
- Exam preparation: 20 h

Total: 75 h (3 ECTS)

Total for the course: 145 h (6 ECTS)

RECOMMENDED READING:

1. Banachowski L., Diks K., Rytter W. : Algorytmy i struktury danych, WNT, W-wa 1996.
2. Koleśnik K.: Wstęp do programowania z przykładami w Turbo Pascalu, Helion, 1999.
3. Sielicki A. (pod red.) : Laboratorium programowania w języku Pascal, Pol. Wr., Wrocław 1996.

OPTIONAL READING:

1. Cormen T.H,. Leiserson Ch.E, Rivest R.L. : Introduction to Algorithms, MIT Press, 2001.
2. Wirth N.: Algorithms and Data Structured, 1985.
3. Kwasowiec W.: Wprowadzenie do Object Pascal i Delphi, MIKOM, 2002.
4. Szmit M.: Delphi, Helion, 2006.

DATABASE SYSTEMS 1

Course code: 11.3-WK-MAT-SP-SBD1
Type of course: optional
Language of instruction: English/Polish
Director of studies: dr inż. Mariusz Hałuszczak
Name of lecturer: prof. dr hab. Mieczysław Borowiecki,
dr Anna Fiedorowicz,
dr inż. Mariusz Hałuszczak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	IV , VI	Exam	
Class	15	1		Grade	
Laboratory	30	2		Grade	

COURSE AIM:

The course introduces basic notions, definitions and problems related to the relational models of databases. At the end of the course each student should be able to design and create both database and database application.

ENTRY REQUIREMENTS:

Fundamentals of logic. Programming skills.

COURSE CONTENTS:

Lecture:

1. The basic notions and definitions related to the relational databases.
2. Operations on relation (union, difference, intersection, complement, projection, selection, join, division).
3. The functional dependencies and Armstrong's axioms.
4. Relational schemes.
5. Decompositions.
6. Normalization through decomposition (1NF, 2NF, 3NF,B-CNF, 4NF, 5NF).
7. Multivalued dependencies.
8. Inference axiom for multivalued dependencies.

Class:

1. Operations on relation
2. Normalization through decomposition (2NF, 3NF,B-CNF).
3. Structured Query Language.

a. Data Manipulation Language,
b. Data Definition Language,
c. Data Control Language.

4. Creating the project of a database.
   a. Data-Flow Diagram,
   b. Entity-Relationship Diagrams,
   c. Creating Database Scheme.

Laboratory:

1. The use of SQL.
2. Data types, expressions and operators, conditions, functions, procedures.
3. SELECT statement:
   a. inner join,
   b. outer join,
   c. simple subqueries,
   d. correlated subqueries,
   e. grouping and aggregate functions.
4. Defining the database structure:
   a. domain,
   b. tables,
   c. views,
   d. indexes,
   e. sequences/generators,
   f. triggers,
   g. referential integrity constraints.
5. Database user management and control of transactions.

TEACHING METHODS:

Lecture: Seminar lecture.

Class: Method problematic, brainstorming.

Laboratory: Computer laboratory exercises.

LEARNING OUTCOMES:

K_W03	Students understand the basic concepts and knows the theoretical basis of relational databases.
K_W08	Students know the basic syntax of SQL commands.
K_W003	Students know the method of normalization of a scheme up to 2NF, 3NF and BCNF.
K_U04	Students are able to extract the information stored in the database using SQL commands, using joins, subqueries and grouping.
K_U28	Students are able to design a simple database schema and generate it using computer tools like CASE.
K_U36	Students are able to present the basic concepts and theorems related to the relational data model.

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Lecture: The exam consists of two parts, written and oral, access to the oral part is getting 30% of the points of the written part, 50% of the points from the written part guarantees a positive evaluation.

Class: condition pass is 50% of the points of the four planned tests or final test covering all the material processed.

Laboratory: condition pass is 50% of the points of the four planned tests or final test covering all the material processed.

Final evaluation of the course is the arithmetic mean of the lecture, class and laboratory. However, a prerequisite for a positive final assessment is to obtain positive evaluations of the lecture, class and laboratory.

STUDENT WORKLOAD:

Contact time with the teacher:

Participation in lectures - 30 hours.

Participation in classes - 15 hours.

Participation in the lab - 30 hours.

Participation in consultations - 18 hours.

Examination - 4 hours.

total: 97 hours.

Standalone student work:

Preparation for the classes - 20 hours.

Preparation for the lab - 20 hours.

Systematic repetition of the material: 6 * 3 hours = 18 hours.

Preparation for the exam - 15 hours.

total: 73 hours.

Total for the course: 170 hours. (6 ECTS)

RECOMMENDED READING:

1. T. Pankowski, Podstawy baz danych, Wydawnictwo Naukowe PWN, W-wa, 1992.
2. D. Maier, The theory of relational databases, Computer Science Press, 1983.
3. M. Gruber, SQL, Helion, 1996.
4. M. Wybrańczyk, Delphi 7 i bazy danych, Helion, 2003.
5. G.Reese, Java. Aplikacje bazodanowe. Najlepsze rozwiązania, Helion, 2003.

OPTIONAL READING:

1. W. Kim, Wprowadzenie do obiektowych baz danych, WNT, Warszawa, 1996.
2. J.D. Ullman, Podstawowy wykład z systemów baz danych, WNT, Warszawa, 1999.
3. P. Neil Gawroński, InterBase dla ,,delfinów'', Helion, 2001.
4. Jakubowski: SQL w InterBase dla Windows i Linuksa, Helion, Gliwice 2001.
5. R. Barker, CASE* Method. Modelowanie związków encji, WNT, Warszawa 2005
6. M. Marzec, JBuilder i bazy danych, Helion, 2005.
7. Mościcki, I. Kruk, Oracle 10g i Delphi. Programowanie baz danych, Helion, 2006.

DATABASE SYSTEMS 1 PROJECT

Course code: 11.3-WK-MAT-SP-SBDP1
Type of course: optional
Language of instruction: English/Polish
Director of studies: dr inż. Mariusz Hałuszczak
Name of lecturer: dr Anna Fiedorowicz,
dr inż. Mariusz Hałuszczak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					4
Project	15	1	V	Grade	

COURSE AIM:

Students design a relational database modelling and create applications for its use.

ENTRY REQUIREMENTS:

Database Systems 1. Programming skills.

COURSE CONTENTS:

Students create a system on a selected topic. Students implement and document the process of creating an information system. The final effect will be a working system, working in a client-server architecture, and documentation.

During the course, students shall analyze the present area, do conceptual data model, SQL script, creating database structure, if it is necessary then create a description of the system using the selected UML diagrams (class, use case, state, activity, implementation), create an application to operate on this database.

Projects are done individually or in groups.

TEACHING METHODS:

Practical

LEARNING OUTCOMES:

K_W08	Students have the theoretical knowledge to design average complex database.
K_W08	Students know how to create an application that supports the database.
K_U29	According to a given specification, students are able to analyze, design and implement a simple database system using properly selected methods, techniques and tools.
K_U25	Students can create an application that supports the database.
	Students can create technical documentation of the project.
K_K03	Students understands necessity of systematic work on the project.

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Total score: 40% for the project database, 40% of the created program, 20% of the documentation.

STUDENT WORKLOAD:

Contact time with the teacher:

Participation in consultations - 2 hours.

Participation in project lessons - 30 hours.

Total: 32 hours.

Standalone student work:

Implementation of project tasks - 30 hours.

Preparation of audit work, reports, reports, etc. - 1 hour.

Total: 31 hours.

Total for all items: 63 hours. (2 credits)

RECOMMENDED READING:

1. D. Maier, The theory of relational databases, Computer Science Press, 1983.
2. M. Wybrańczyk, Delphi 7 i bazy danych, Helion, 2003.
3. M. Marzec, JBuilder i bazy danych, Helion, 2005.
4. G.Reese, Java. Aplikacje bazodanowe. Najlepsze rozwiązania, Helion, 2003.
5. Mościcki, I. Kruk, Oracle 10g i Delphi. Programowanie baz danych, Helion, 2006.

OPTIONAL READING:

1. P. Neil Gawroński, InterBase dla ,,delfinów'', Helion, 2001.
2. Jakubowski: SQL w InterBase dla Windows i Linuksa, Helion, Gliwice 2001.
3. R. Barker, CASE* Method. Modelowanie związków encji, WNT, Warszawa 2005

DATABASES SYSTEMS 2

Course code: 11.3-WK-MAT-SP-SBD2
Type of course: optional
Language of instruction: English/Polish
Director of studies: dr inż. Mariusz Hałuszczak
Name of lecturer: prof. dr hab. Mieczysław Borowiecki, dr inż. Mariusz Hałuszczak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	V	Exam	
Class	15	1		Grade	
Laboratory	30	2		Grade	

COURSE AIM:

Familiarize students with the ORACLE database and PL/SQL. At the end of the course students should be able to independently design and develop dynamic website using database.

ENTRY REQUIREMENTS:

Programming skills. Basic knowledge of relational databases and SQL, and HTML.

COURSE CONTENTS:

Lecture:

1. PL/SQL
   - Structure of programme, variables, data types, expressions, comparisons and control structures.
   - Using collections and records.
   - Performing SQL operations from PL/SQL.
   - Procedures, functions, packages in PL/SQL.
   - Handling PL/SQL errors.
   - Dynamic PL/SQL.
2. Acyclic database.
3. Deriving functional dependencies using:
   - Armstrong’s axioms.
   - B-axioms
   - directed acyclic graphs.

Class:

1. PHP
   - Operations on text and numbers.

- Creating interactive forms.
- Communication with databases.
- Sessions and cookies.
- File operations.

2. Object-oriented databases and XML

   Structure of the XML document.

   DTD and XML-Schema.

   XSLT.

1. JavaScript
   - Syntax,
   - Classes, objects, JSON format,
   - AJAX and XML support,
   - Examples of frameworks.

Laboratory:

1. SQL in Oracle.
2. Tree structures in Oracle databases.
3. PL/SQL, create stored procedures, functions, triggers, and packages.
4. Views describes schema objects in the database.
5. Transforming XML data with XSLT and JavaScript.

TEACHING METHODS:

Lecture: Seminar lecture.

Class: Method problematic, brainstorming, presentations.

Laboratory: Computer laboratory exercises.

LEARNING OUTCOMES:

K_W08	Students know the syntax of SQL commands and PL/SQL.
K_W03	Students are able to extract and present data that are stored in XML format.
K_U27	Students can collect and extract the information stored in databases with web applications.
K_K06	Students can search for relevant information.

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Lecture: The exam consists of two parts, written and oral, access to the oral part is getting 30% of the points of the written part, 50% of the points from the written part guarantees a positive evaluation.

Class: condition pass is 50% of test covering all the material processed.

Laboratory: condition pass is 50% of the points of the four planned tests or final test covering all the material processed, for the preparation of the talk, you can get up to an additional 20%..

Final evaluation of the course is the arithmetic mean of the lecture, class and laboratory. However, a prerequisite for a positive final assessment is to obtain positive evaluations of the lecture, class and laboratory.

STUDENT WORKLOAD:

Contact time with the teacher:

Participation in lectures - 30 hours.

Participation in classes - 15 hours.

Participation in the lab - 30 hours.

Participation in consultations - 18 hours.

Examination - 4 hours.

Total: 97 hours.

Standalone student work:

Preparation for the classes - 20 hours.

Preparation for the lab - 30 hours.

Preparation for the Exam - 20 hours.

Reading the literature - 10 hours.

Total: 80 hours.

Total for the course: 177 hours. (6 ECTS)

RECOMMENDED READING:

1. D. Maier, The theory of relational databases, Computer Science Press, 1983.
2. E. Balanescu, M. Bucica, Cristian Darie, PHP 5 i MySQL. Zastosowania e-commerce, Helion, 2005.
3. J. Clark, XSL Transformations (XSLT), http://www.w3.org/TR/xslt7.
4. L. Quin, Extensible Markup Language (XML), http://www.w3.org/xml.
5. T. Converse, J. Park, C. Morgan, PHP5 i MySQL. Biblia, Helion, 2005.
6. S. Urman, R. Hardman, M. McLaughlin, Oracle Database 10g. Programowanie w języku PL/SQL, Helion, 2007.

OPTIONAL READING:

1. E. Naramore, J. Gerner, Y. Le Scouarnec, J. Stolz, M.K. Glass, PHP5, Apache i MySQL. Od podstaw, Helion, 2005.
2. B. Basham, K. Sierra, B. Bates, Head First Servlets & JSP, Helion, 2005.
3. W. Kim, Wprowadzenie do obiektowych baz danych, WNT, Warszawa, 1996.

DATABASES SYSTEMS 2 PROJECT

Course code: 11.3-WK-MAT-SP-SBDP2
Type of course: optional
Language of instruction: English/Polish
Director of studies: dr inż. Mariusz Hałuszczak
Name of lecturer: dr inż. Mariusz Hałuszczak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					4
Project	15	1	VI	Grade	

COURSE AIM:

Students design a relational database model and create WEB applications for its use.

ENTRY REQUIREMENTS:

Databases Systems 2.

COURSE CONTENTS:

Students create a system on a selected topic. Students implement and document the process of creating an information system. The final effect will be a working WEB application, working in a client-server architecture, and documentation.

During the course, students shall analyze the present area, do conceptual data model, SQL script, creating database structure, if it is necessary then create a description of the system using the selected UML diagrams (class, use case, state, activity, implementation), create an application to operate on this database.

Projects are done individually or in groups.

TEACHING METHODS:

Practical

LEARNING OUTCOMES:

K_W08	Students have the theoretical knowledge to design average-complex database.
K_W08	Students know how to create a WEB application that supports the database.
K_U29	Students can, in accordance with specifications, analyze, design and implement a simple database system using properly selected methods, techniques and tools.
	Students can create technical documentation of the project.
K_K03	Students understand the need for systematic work on the project.

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Total score: 40% for the project database, 40% of the created program, 20% of the documentation.

STUDENT WORKLOAD:

Contact time with the teacher:

Participation in project lessons - 15 hours.

Participation in consultations - 20 hours.

Total: 35 hours.

Standalone student work: 65 hours.

Total for all items 100 hours. (4 ECTS)

RECOMMENDED READING:

1. E. Balanescu, M. Bucica, Cristian Darie, PHP 5 i MySQL. Zastosowania e-commerce, Helion, 2005.
2. J. Clark, XSL Transformations (XSLT), http://www.w3.org/TR/xslt7.
3. L. Quin, Extensible Markup Language (XML), http://www.w3.org/XML.
4. T. Converse, J. Park, C. Morgan, PHP5 i MySQL. Biblia, Helion, 2005.
5. S. Urman, R. Hardman, M. McLaughlin, Oracle Database 10g. Programowanie w języku PL/SQL, Helion, 2007.
6. E. Naramore, J. Gerner, Y. Le Scouarnec, J. Stolz, M.K. Glass, PHP5, Apache i MySQL. Od podstaw, Helion, 2005.

OPTIONAL READING:

1. Bryan Basham, Kathy Sierra, Bert Bates, Head First Servlets & JSP. Helion, 2005
2. Wojciech Romowicz, Java Server Pages oraz inne komponenty JavaPlatform, Helion, 2001

DIFFERENTIAL EQUATIONS

Course code: 11.1-WK-MAT-SP-RR
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Tomasz Małolepszy
Name of lecturer: dr Tomasz Małolepszy

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	IV	Exam	
Class	30	2		Grade	

COURSE AIM:

The main aim of this course is to familiarize students with the basic theory of ordinary differential equations: finding solutions of first-order and second-order ODE as well as first-order systems of ODE, the existence and the uniqueness of solutions of ODE, testing stability of critical points and making phase portraits of linear system in the plane.

ENTRY REQUIREMENTS:

Mathematical Analysis 1 and 2, Linear Algebra 1 and 2.

COURSE CONTENTS:

Lecture

1. Basic concepts: n-order ordinary differential equation, system of *n*-order ordinary differential equations, solution of ODE, continuation of solution, saturated solutions, general and particular solution, integral curves, first integrals, phase space. (2 hours)
2. First-order ordinary differential equations.
   Examples of phenomena leading to ODE. Geometrical interpretation of ODE. Separable equations and the types of equations reducible to separable equations. Linear ODE and equations reducible to linear equations (Bernoulli equations, Riccati equations). Exact equations. (5 hours)
3. Existence and uniqueness of local solutions of initial problems for ODE.
   Cauchy problem for ODE. Picard-Lindelöf theorem. Gronwall's lemma. Peano theorem. Extension of solutions of the initial problems for ODE. Dependence of the solution to Cauchy problem on initial conditions and the right-hand side of the equation. (6 hours)
4. Second-order ordinary differential equations.
   Physical motivation. Types of equations reducible to first-order ordinary differential equations. Linear second-order differential equations. (4 hours)
5. Systems of linear first-order differential equations.
   Existence and uniqueness of solutions. Systems of homogeneous equations, fundamental matrix. Systems of homogeneous equations with constant coefficients. Systems of inhomogenous equations. (6 hours)

6. Elements of the qualitative theory of ODE.

   Classification and stability of critical points of systems of linear ODE in the plane. Phase portraits of linear systems. Classification and stability of critical points of systems of nonlinear ODE in the plane. Lyapunov stability. Lyapunov function and fundamental stability theorems. (7 hours)

Class

1. Solving first-order ordinary differential equations: separable equations and the types of equations reducible to separable equations, linear equations, Bernoulli equations, Riccati equations, exact equations. Solving exercises related to physical phenomena which should be described in terms of Cauchy problems for ODE. (8 hours)
2. Solving exercises with the use of existence and uniqueness theorems of local solutions of initial problems for ODE. (3 hours)
3. Test. (2 hours)
4. Solving second-order ordinary differential equations by reducing them to first-order ordinary differential equations. Solving second-order linear ordinary differential equations. (5 hours)
5. Solving systems of first-order linear ordinary differential equations - computation of fundamental matrix. (5 hours)
6. Examination of stability of critical points of systems of first-order linear ODE. Sketching phase portraits. (5 hours)
7. Test. (2 hours)

TEACHING METHODS:

Traditional lectures; classes with the lists of exercises to solve by students.

LEARNING OUTCOMES:

Student is able:

1. to solve such I order ordinary differential equations as separable equations, linear equations, exact differential equations as well as II order linear equations with constant coefficients, (K_U01+, K_W05+)
2. to interpret a system of ordinary differential equations in the language of geometry by means of vector field and phase space, (K_U22+++)
3. to formulate Peano Theorem as well as Picard-Lindelöf Theorem, (K_W04+)
4. to describe basic physical phenomena such as a radioactive decay or the cooling of object in terms of Cauchy problems for I order ODEs, (K_W01+)
5. to solve some basic linear systems of ODEs and to sketch phase plane portraits of such systems. (K_U01+, K_U21++, K_W05+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Class: learning outcomes will be verified through two tests consisted of exercises of different degree of difficulty. A grade determined by the sum of points from these two tests is a basis of assessment.

Lecture: final exam. A grade determined by the sum of points from that exam is a basis of assessment.

A grade from the course is consisted of the grade from classes (40%) and the grade from the final exam (60%). To take a final exam, students must receive a positive grade from classes. To attain a pass in the course students are required to pass the final exam.

STUDENT WORKLOAD:

Contact hours

Lectures - 30 hours.

Classes - 30 hours.

Lectures’ consultation hours - 10 hours.

Classes’ consultation hours - 10 hours.

Total - 80 hours (3 ECTS).

Individual work

Preparation to lectures - 5 hours.

Preparation to classes - 35 hours.

Preparation to the final exam - 30 hours.

Total - 70 hours (3 ECTS).

Total time needed for this course: 150 hours (6 ECTS).

RECOMMENDED READING:

1. Andrzej Palczewski, *Równania różniczkowe zwyczajne*, WNT, Warszawa 1999.
2. Władimir I. Arnold, *Równania różniczkowe zwyczajne*, PWN, Warszawa 1975.
3. William E. Boyce, Richard C. DiPrima, *Elementary differential equations and boundary value problems*, Wiley, New York 2001.

OPTIONAL READING:

1. Marian Gewert, Zbigniew Skoczylas, *Równania różniczkowe zwyczajne. Teoria, przykłady, zadania*, Oficyna Wydawnicza GiS, Wrocław 2008.

DISCRETE MATHEMATICS 1

Course code: 11.1-WK-MAT-SP-MD1
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr hab. Ewa Drgas-Burchardt
Name of lecturer: prof. dr hab. Mieczysław Borowiecki, dr hab. Ewa Drgas-Burchardt

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					5
Lecture	30	2	II	Exam	
Class	30	2		Grade	

COURSE AIM:

The course introduces basic notions and ideas of the discrete mathematics in theoretic and algorithmic aspects.

ENTRY REQUIREMENTS:

Introduction to mathematics, Linear algebra 1

COURSE CONTENTS:

Lecture

1. Basic notions of the graph theory: neighbourhood, adjacency, isomorphism, paths, cycles, connectivity, subgraphs (2 h).
2. Graph matrixes (2 h).
3. Some classes of graphs (2 h).
4. Union, join and complement graph operations (2 h).
5. Trees and their properties (4 h).
6. BFS and DFS algorithms (2 h).
7. Vector spaces of the graph (2 h).
8. *n*-connectivity (2 h).
9. Eulerian graphs. Hamiltonian Graphs (3 h).
10. Planar graphs, Kuratowski's Theorem, Harary's Theorem (3 h).
11. Covers and independence (2 h).
12. Vertex colouring of graphs, list colouring of graphs, Brooks's Theorem, the Szekeres-Wilf Theorem, Vizing's Theorem, Thomassen's Theorem (4 h).

Class

1. Reading information on a graph from its matrixes, adjacency lists, sets of pairs. Interpretation of operations on graph matrixes. Matrixes of graph operations (4 h).

2. Investigation of basic tree's features. Counting labeled trees, using known algorithms to find a spanning tree of a graph, its sets of fundamental cycles and elementary cut. Generating of cycle and cut spaces of a graph. Construction of a modular decomposition tree of a graph (8 h).
3. Analysis of graph connectivity (2 h).
4. Investigation of Eulerian and Hamiltonian graphs and dependence of these properties on other features of graphs. Using known algorithms to recognize an Euler tour and a Hamilton cycle in a graph (4 h).
5. Recognition of problems associated with graph planarity, independence and covers numbers of a graph in practical exercises. Application of theoretical knowledge in practical problems on this topic (4 h).
6. Recognition of coloring problems in practice. Theoretical and algorithmic approach (6 h).
7. Test completion (2 h).

TEACHING METHODS:

Conversation lecture, traditional lecture, discussion exercises

LEARNING OUTCOMES:

1. A student is able to name and define the basic concepts of discrete mathematics (K_W06).
2. A student is able to perform simple proofs in the field of discrete mathematics (K_W02, K_U01, K_U04).
3. A student is able to name linear spaces associated with a graph (K_U17).
4. A student is able to apply an appropriate algorithm, among those which were presented at the course, to solve the problem of discrete mathematics (K_U25).
5. A student is able to decide with which objects in the field of discrete mathematics the solution of the practical problem can be identified (K_U29).
6. A student understands the significance of intellectual honesty, both in their own and in other people's activities; demonstrate ethical behavior (K_K04).

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Methods:
D - participation in the discussions during the course
P1 - essay
P2 - written exam
PU2 - oral exam
S - self-esteem

Assessment of individual classes:

1. Checking of preparedness of students and their activity during exercise (D, S).
2. Colloquium with the tasks of different difficulty, allowing to evaluate whether the students have achieved specified learning outcomes in terms of skills and competencies (P1).
3. Conversation during the lecture in order to verify the effects of higher levels of education in terms of knowledge and skills (D, S).
4. Written exam to verify the learning outcomes in terms of knowledge and skills (P2).
5. Oral exam, which allows to complete student's written expression (PU2).

The grade of the module consists of the assessment exercise (50%), exam grade (P2 + PU2) (50%). The condition of the exam is to get a positive assessment of the exercise. The prerequisite to obtain a positive evaluation of the module is the positive evaluation of the exercise and the exam.

STUDENT WORKLOAD:

Activity	Student load
Participation in lectures	30 h
Participation in exercises	30 h
Self preparation for lectures	30 h
Independent problem solving	30 h
Consultation	10 h
The combined student workload	130 h

Number of ECTS credits allocated 5

RECOMMENDED READING:

1. V. Bryant, Aspekty kombinatoryki, WNT, Warszawa, 1997.
2. W. Lipski, Kombinatoryka dla programistów, WNT, Warszawa, 2005.
3. K.A. Ross, Ch.R.B. Wright, Matematyka dyskretna, PWN, Warszawa 1996.
4. R. J. Wilson, Wprowadzenie do teorii grafów, PWN, Warszawa, 1998.
5. D. West, Introduction to Graph Theory, 2nd ed., Prentice Hall, Upper Saddle River, 2001.

OPTIONAL READING:

1. W. Lipski, W. Marek, Analiza kombinatoryczna, PWN, Warszawa, 1989.

ELEMENTARY GEOMETRICS

Course code: 11.1-WK-MAT-SP-GE
Type of course: eligible
Language of instruction: English/Polish
Director of studies: dr Krystyna Białek, dr hab. Krzysztof Przesławski, prof. UZ
Name of lecturer: dr Krystyna Białek, dr Andrzej Kisielewicz

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					
Lecture	30	2	VI	Exam	5
Class	30	2		Grade	

COURSE AIM:

Introduction with basic notions, methods of elementary geometry as well as the students' equipment in basic mathematical indispensable tools to formulating and solving typical, the straight lines of tasks and the problems with range of studied direction of studies.

ENTRY REQUIREMENTS:

Elementary algebra, linear algebra, basic analytic geometry.

COURSE CONTENTS:

Lecture

1. Isometrics of Euclidean plane: definitions, examples, kinds, classification of isometrics of plane – 2h.
2. The similarity of Euclidean plane: basic definitions, dilatations, classification of similarities of plane - 2h.
3. Affine transformation: basic definitions, property, the analytic figure of affine transformation, matrices criteria - 2h.
4. Points and lines associated with a triangle: Menelaus' and Ceva's theorems-2h.
5. The Euler line and the 9-point circle - 2h.
6. The power of point with respect to a circle-2h.
7. The Theorems of Euler. The power straight line of steam of circles. The power centre of three of circles. The Brianchon Theorems -2h.
8. Circle inversion. The Feuerbach's Theorems – 2h.
9. Geometric constructions. Constructional problems, methods of solving the constructional problems. Constructions using ruler and compass - 2h.
10. The impossibility of solving the tree Famous Problems of Antiquity with Euclidean Tools - 2h.
11. Constructions of regular polygons. The constructions of chosen of regular polygons - 2h.
12. Constructions unclassic centres. The Mohr– Marcheroni Construction Theorem. The Poncelet-Steiner Construction Theorem – 2h.
13. Convex Polyhedron, Euler's formula, Platonian clods -2h.

14. An axiomatic approach to Euclidean geometry, absolute geometry. Various formulations of the fifth postulate -2h.
15. Hyperbolic geometry and its models (Klein and Poincaré models). Basic theorems. Other non-Euclidean geometries -2h.

Class

1. Geometric transformations of Euclidean plane: isometrics, method of building the groups of transformations, examples of groups - 2h.
2. Isometrics on plane: analytic formulas central symmetry, translation, axial symmetry, turn around point about directed angle; the group of Isometric of own figures; examples - 2h.
3. Similarity of Euclidean plane: characteristics, examples and classification - 2h.
4. Affine Transformation: analytic figure, matric criteria - 2h.
5. Points and lines associated with a triangle: the Theorems Menelaus and Ceva, Applications of the Theorem of Menelaus and Ceva - 2h.
6. The Euler line and the 9-point circle - geometrical interpretation-2h.
7. Power of a point with respect to a circle – 2.
8. Theorems concerning chords, secants and tangents of a circle. Radical axis of two circles – 2h.
9. Circle inversion. Orthogonal circles- 2h.
10. Interim control written test - 2h.
11. Geometric constructions. Constructional problems, methods of solving the constructional problems. Constructions using the ruler and compass - 2h.
12. The impossibility of solving the tree Famous Problems of Antiquity with Euclidean Tools - 2h.
13. Constructions of regular polygons. The constructions of chosen of regular polygons - 2h.
14. Convex Polyhedron, Eulera's theorem, Platonian clods - 2h.
15. Written test -2h.

TEACHING METHODS:

Lecture: conventional, problematic, introduction.
Practice: the classic problematic method, work in groups, the demonstration from explanation, the discussion, storm of brains, work with programme C and R. in computer laboratory

LEARNING OUTCOMES:

1. Know basic theorems of Euclidean Plane Isometrics and understand the presence of the group structure of the transformations and build a group structure of the transformations K_W04 K_U17, K_K07.
2. Know basic of Euclidean Plane similarity and is able to see the presence of the group structure of the transformations and build a group structure of the transformations K_W04 K_U17, K_K07
3. Know basic theorems of Euclidean Plane Isometrics and understand the presence of the group structure of the transformations and build a group structure of the transformations K_W04 K_U17, K_K07.
4. Know basic theorems of triangle geometry with their proves and is able to use concept of vector in proving the Menelaos and Cevy theorem. Deepen their knowledge and abilities relating to the scope of their interests; are able to obtain information from specialist literature independently, also in foreign languages K_U 16, K_K 06.
5. Understands relation between the algebraic and geometrical description of transformations and algebraic sets the first and second level and is able to formulate statements and definitions in the intelligible way in the speech and the letter, is able to formulate opinions about the description of transformations and algebraic sets at the first and second level K_W 01, K_U 01, K_K 07.
6. Understands the role and meaning of proof in geometric structures, as well as significance of assumptions in solving structural problems and is able to build basic geometric structures with compasses and the ruler as well as in free access computer programs (e.g. C.a.R) K_W 02, K_U 01, K_K 01.
7. Know methods of axiomatic in geometry and is able to perceive the elementary geometry as the example of the axiomatic theory and the classic system of the deduction including the historical development of geometrical problems, he is able to formulate statements and definitions based on the contemporary system of axioms of the Euclidean plane geometry, is able to search independently for information in literature, also in foreign languages, K_W 03 K_U 06, K_K 07.

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Assessment form - the final written exam.
The final grades: average grade of tests and exam.
The condition of positive assessment of practices - the positive assessment of two tests as well as the activity on practices.

To pass the test you need to obtain settled (for given test / colloquium) the minimum number of points (50%).
The condition of positive assessment of examination - the positive assessment of multiple choice test (the examples illustrated the lecture) by obtaining the settled minimum number of points of test (50%).
Checking students' preparation to classes as well as their activity on the practices.
The tests (colloquia) include tasks of diverse level of difficulty, permitting to assess if a student has reached the learning outcomes on basic level.

STUDENT WORKLOAD:

Contact hours :
lectures – 30h
practices – 30h.
consultations: 5h to lecture + 5h to practices = 10h.
Together: 70h. (2 ECTS)
Student work:
preparation to lecture – 20h
preparation to practices – 40h
preparation to examination – 30h
Together: 90h (3 ECTS)
Together for whole object: 160h (5 ECTS)

RECOMMENDED READING:

1. Aleksandrow I. I.: *Zbiór geometrycznych zadań konstrukcyjnych*, PZWS, Warszawa 1964
2. Borsuk K., Szmielew W.: *Podstawy geometrii,.* PWN, Warszawa 1970
3. Doman R.: *Wykłady z geometrii elementarnej,* Wyd. Naukowe UAM, Poznań 2001
4. Kordos,M. Szczerba L., W.: *Geometria dla nauczycieli,* PWN, Warszawa 1976
5. Coxeter S. M.: *Wstep do geometrii dawnej i nowej,* PWN, Warszawa 1967
6. Kowalski E.: *Geometria dla studentów,* WSP, Zielona Góra 1990
7. Modenov P.: Parhomenko A.: *Geometric Transformations.* Acad. Press, New York, 1965
8. Szmielew W.: *Od geometrii afinicznej do euklidesowej*, PWN, 1983
9. Zetel S. I.: *Geometria trójkąta,* PZWS, Warszawa 1964

OPTIONAL READING:

1. Berger M.: *Geometrie,* Nathan, Paris 1977
2. Coxter H.S.,M, Greitzer S.,L.: *Geometry revisited,* Toronto New York 1967
3. Neugebauer A.: *Wstęp do planimetrii,* Wydawnictwo Naukowe US, Szczecin 2000

ELEMENTS OF THE HISTORY AND PHILOSOPFY OF MATHEMATICS

Course code: 11.1-WK-MAT-SP-EHFM
Type of course: optional
Language of instruction: polish
Director of studies: prof. dr hab. Marian Nowak
Name of lecturer: prof. dr hab. Marian Nowak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Lecture	30	2	IV or VI	Pass/Fail	

COURSE AIM:

Students should be familiar with an outline of history of main fields of mathematics, in particular, with evolution of the most important concepts and ideas in mathematics . Moreover , students should know the main directions in modern philosophy of mathematics.

ENTRY REQUIREMENTS:

Standard graduate courses in : logic and the set theory, geometry , mathematical analysis.

COURSE CONTENTS:

1. Outline of history of logic and the set theory. (2 hours)
2. Foundations of mathematics. Formalized theories. Hilbert's program . Models of mathematical theories. Godel's theorems and their philosophical implications. (2 hours)
3. The classical directions in the modern philosophy of mathematics. Platonism, formalism, intuitionism, constructivism. (2 hours)
4. The problem of the truth and existence in mathematics. The cultural basis of mathematics. (2 hours)
5. Different concepts of reconstruction of mathematics. Reconstruction of mathematics on the base of the set theory. Structuralism in mathematics: Bourbaki. Theory of categories (2 hours)
6. Elements of history of mathematics.
   Mathematics in the Ancient Orient.(2 hours)
   Mathematics in Greece.(2 hours)
   Arabic Mathematics (2 hours)
   Mathematics in XVI century.(2 hours)
   Mathematics in XVII century.(2 hours)
   Mathematics in XVIII century.(2 hours)
   Mathematics in XIX century.(2 hours)
7. Evolution of the most important mathematical concepts and ideas. Outline of history of geometry, algebra and mathematical analysis.(4 hours)
8. Outline of history of mathematics in Poland. The Lvov and the Warsaw mathematical schools.(2 hours).

TEACHING METHODS:

Traditional lecture , open to a discussion and expressing opinions by students.

LEARNING OUTCOMES:

1. A student can characterize axiomatic-deductive structure of mathematical theories. He can describe the concept of a model of a mathematical theory.(KW03++)
2. Student is familiar with an evolution of basic mathematical concepts , in particular , the concepts of number and space. He/she is familiar with an outline of evolution of basic fields in mathematics: geometry, algebra and mathematical analysis.(KW07++)
3. Student knows and can describe the classical directions of modern philosophy of mathematics.(KU07++)
4. He/she can characterize position and importance of Polish mathematics (KW01++)
5. Student can work individually with a source literature on history and philosophy of mathematics.(KK06+)
6. Student can explain the civilizing and cultural meaning of mathematics.(KW01+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Students write an essay on a chosen subject in the history or philosophy of mathematics.

STUDENT WORKLOAD:

Contact hours

Lecture - 30 hours

Office hours - 5 hours

Individual work

Preparation of an essay – 25 hours

Entire subject jointly : 60 hours (2 ECTS)

RECOMMENDED READING:

1. D. Struik, A Concise History of Mathematics, Dover Publications ,Inc., New York,1948.
2. R. Murawski, Filozofia matematyki. Zarys dziejów, Warszawa 1995.
3. P. Davis, R. Hersh, Świat matematyki, PWN , Warszawa 1994.
4. James, Remarkable Mathematicians, Cambridge University Press 2002.

OPTIONAL READING:

1. M. Kordos, Wykłady z historii matematyki, Wyd. II, SCRIPT, Warszawa 2006.
2. M. Murawski, Współczesna filozofia matematyki, Wybór tekstów, Wydawnictwo Naukowe PWN , Warszawa 2002.

ENGLISH 1

Course code: 09.0-WK-MAT-SP-JA1
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: mgr Grażyna Czarkowska
Name of lecturer: mgr Grażyna Czarkowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Laboratory	30	2	II	Grade	

COURSE AIM:

The course aims to enable students to improve speaking, reading and writing skills, as well as listening comprehension in English. It will help the students to develop their ability to apply language functions to effective communication in everyday life. The course also aims to further develop ability to use grammar structures which describe present and past activities and are used to form questions.

The course will introduce elements of the language of mathematics – basic vocabulary used in number theory, and expressions used to describe basic operations in mathematics.

ENTRY REQUIREMENTS:

A2 of the Common European Framework of Reference for Languages specified by the Council of Europe

COURSE CONTENTS:

During the course students will learn how to:

- describe present and past activities using appropriate grammar tenses (8 hours)
- form basic questions in English - question words and auxiliary verbs (2 hours)
- exchange and get information in everyday life situations (3 hours)
- have a simple conversation in English (3 hours)
- read and understand texts describing present and past (4 hours)
- develop listening comprehension (2 hours)
- express opinions on social phenomena in a discussion in English (2 hours)
- read numbers – ordinal, cardinal, fractions (common, decimal)
- read dates and mathematical operations (4 hours)
- read with understanding simple mathematical texts concerning number theory (2 hours)

TEACHING METHODS:

The course focuses on communication activities in functional and situational context. It encourages students to speak with fluency and develop the four skills of reading, writing, listening and speaking by means of group and pair work, discussion , presentation, oral and written exercises.

LEARNING OUTCOMES:

Achieving language skills and competence on level A2+ of the Common European Framework of Reference for Languages.

Upon successful completion of the course, the students:

- are able to describe present and past activities using simple grammar structures-tenses
- can form simple questions in English
- give basic information concerning everyday life – personal data, habits, preferences
- are able to get information concerning everyday life
- are able to have simple conversations
- understand non-specialist texts describing present and past activities
- can read numbers – cardinal, ordinal, fractions
- can read dates and basic mathematical operations
- understand simple specialist texts concerning number theory
- are able to work in a team

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Classes – grade: a condition for receiving a credit are positive marks for tests, taking part in discussions and giving a short presentation in English.

STUDENT WORKLOAD:

Contact time:

- classes – 30 hours
- consultation – 5 hours

Private study – 25 hours

RECOMMENDED READING:

1. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Student's Book,* Oxford University Press 2007
2. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Workbook,* Oxford University Press 2007

OPTIONAL READING:

1. *FCE Use of English* by V. Evans
2. Internet articles
3. L. Szkutnik, *Materiały do czytania – Mathematics, Physics, Chemistry,* Wydawnictwa Szkolne i Pedagogiczne
4. J. Pasternak-Winiarska, *English in Mathematics*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2006

ENGLISH 2

Course code: 09.0-WK-MAT-SP-JA2
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: mgr Grażyna Czarkowska
Name of lecturer: mgr Grażyna Czarkowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Laboratory	30	2	III	Grade	

COURSE AIM:

The course aims to enable students to improve speaking, reading and writing skills, as well as listening comprehension in English. It will help the students to develop their ability to apply language functions to effective communication in everyday life. The course also aims to further develop ability to use grammar structures which describe future and life experiences. It will help students to revise structures used to talk about present and past. The course provides an opportunity to learn the skill of writing informal letters.

The students will be able to deepen their knowledge of mathematical language used in number theory.

ENTRY REQUIREMENTS:

A2+ of the Common European Framework of Reference for Languages specified by the Council of Europe

COURSE CONTENTS:

During the course students will learn to:

- describe present and past activities using more complex language structures - continuous tenses (2 hours)
- describe future activities – predictions, plans (4 hours)
- express offers, suggestions (2 hours)
- talk about life experiences using appropriate grammar tense (4 hours)
- exchange and get information concerning future in everyday life situations (3 hours)
- have longer conversations using familiar vocabulary and language structures (3 hours)
- understand non-specialist texts describing future (4 hours)
- participate in class discussions, express opinions with confidence (2 hours)
- write informal letters (2 hours)
- improve listening comprehension (2 hours)
- master and extend vocabulary used in number theory
- give definitions of natural, rational, irrational, real and complex numbers (2 hours)

TEACHING METHODS:

The course focuses on communication activities in functional and situational context. It encourages students to speak with fluency and develop the four skills of reading, writing, listening and speaking by means of group and pair work, discussion, presentation, oral and written exercises.

LEARNING OUTCOMES:

Achieving language skills and competence on level A2+ of the Common European Framework of Reference for Languages.

Upon successful completion of the course, the students:

- are able to describe present and past activities using complex grammar structures and recognize situational context for their application
- are able to describe life experience using appropriate grammar tenses
- are able to express offers and suggestions
- are able to get detailed information concerning everyday life.
- can have longer conversations using more complex structures and vocabulary
- understand non-specialist texts describing future
- have developed listening comprehension to understand longer dialogues
- know expressions and rules used in informal letters
- are able to give simple definitions of natural, rational, etc. numbers
- are able to get information about topics from number theory
- are able to work in a team

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Classes – grade: a condition for receiving a credit are positive marks for tests, participating in class discussions, dialogues, delivering a presentation in English, getting information.

STUDENT WORKLOAD:

Contact time:

- classes – 30 hours
- consultation – 5 hours

Private study – 25 hours

RECOMMENDED READING:

1. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Student's Book,* Oxford University Press 2007
2. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Workbook,* Oxford University Press 2007

OPTIONAL READING:

1. *FCE Use of English* by V. Evans
2. L. Szkutnik, *Materiały do czytania – Mathematics, Physics, Chemistry,* Wydawnictwa Szkolne i Pedagogiczne
3. Internet articles
4. J. Pasternak-Winiarska, *English in Mathematics*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2006

ENGLISH 3

Course code: 09.0-WK-MAT-SP-JA3
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: mgr Grażyna Czarkowska
Name of lecturer: mgr Grażyna Czarkowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Laboratory	30	2	IV	Grade	

COURSE AIM:

The course aims to enable students to improve speaking, reading and writing skills, as well as listening comprehension in English. It will help the students to develop their ability to apply language functions to effective communication in everyday life. The course also aims to develop ability to compare objects, people, phenomena, to express necessity, prohibition and orders. The course provides an opportunity to learn the skill of writing formal letters, improve listening and reading comprehension. It helps students to further develop conversational skills, and gives basic knowledge of giving a presentation in English. It introduces vocabulary used in predicatie logic.

ENTRY REQUIREMENTS:

B1 of the Common European Framework of Reference for Languages specified by the Council of Europe.

COURSE CONTENTS:

During the course students will learn to:

- compare people, objects (4 hours)
- use modal verbs to express prohibition and orders (3 hours)
- write formal letters (4 hours)
- use verb forms – gerund, infinitive (3 hours)
- make a longer dialogue using structures and vocabulary learned earlier in the course – comparison, modals to express prohibition, etc. (2 hours)
- prepare and deliver a short presentation in English (4 hours)
- understand longer and more difficult texts (2 hours)
- develop listening comprehension of long conversations (2 hours)
- master vocabulary of logic and set theory (4 hours)
- understand simple specialist texts concerning logic (2 hours)

TEACHING METHODS:

The course focuses on communication activities in functional and situational context. It encourages students to speak with fluency and develop the four skills of reading, writing, listening and speaking by means of group and pair work, discussion, presentation, oral and written exercises.

LEARNING OUTCOMES:

Achieving language skills and competence on level B1+ of the Common European Framework of Reference for Languages.

Upon successful completion of the course, the students:

- can compare people, objects, and phenomena
- can express prohibition, orders using modal verbs
- are able to write formal letters
- use verb forms (gerund, infinitive) according to the rules
- have long dialogues using complex language structures and vocabulary
- are able to deliver a short presentation on a chosen topic in mathematics
- are familiar with vocabulary used in logic
- understand specialist texts concerning logic
- know the definition of sets, kinds of sets and operations on sets
- can cooperate with members of a group, exchange information, and discuss problems

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Classes – grade: a condition for receiving a credit are positive marks for tests, participating in class discussions, dialogues, delivering a presentation in English, getting information on different topics.

STUDENT WORKLOAD:

Contact time:

- classes – 30 hours
- consultation – 5 hours

Private study – 25 hours, students systematically prepare for the examination.

RECOMMENDED READING:

1. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Student's Book,* Oxford University Press 2007
2. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Workbook,* Oxford University Press 2007
3. J. Pasternak-Winiarska, *English in Mathematics*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2006

OPTIONAL READING:

1. *FCE Use of English* by V. Evans
2. L. Szkutnik, *Materiały do czytania – Mathematics, Physics, Chemistry*, Wydawnictwa Szkolne i Pedagogiczne
3. Internet articles
4. R. Murphy *English Grammar in Use*.

ENGLISH 4

Course code: 09.0-WK-MAT-SP-JA4
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: mgr Grażyna Czarkowska
Name of lecturer: mgr Grażyna Czarkowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Laboratory	30	2	V	Exam	

COURSE AIM:

The course aims to enable students to improve speaking, reading and writing skills, as well as listening comprehension in English. It will help the students to develop their ability to apply language functions to effective communication in everyday life. The course also aims to develop ability to describe hypothetical situations, express probability, give advice and use Passive Voice properly. The course provides an opportunity to learn the skill of writing formal letters, improve listening and reading comprehension. It helps students to further develop conversational skills, and ability to deliver a presentation in English.

The course helps students to develop vocabulary from the following branches of mathematics: geometry, integral calculus, differential calculus, derivatives.

ENTRY REQUIREMENTS:

B1+ of the Common European Framework of Reference for Languages specified by the Council of Europe.

COURSE CONTENTS:

During the course students will learn to:

- describe hypothetical situations, use conditional sentences referring to present, future and past (6 hours)
- use clauses of time introduced by *when, as soon as, till, before, after* (2 hours)
- use modal verbs to express probability (1 hour)
- understand and form correct sentences in Passive Voice (4 hours)
- understand long and difficult non-specialist texts describing hypothetical situations, as well as discussing social issues (5 hours)
- prepare and deliver a presentation in English using language structures studied during the course (6 hours)
- develop listening skills (2 hours)
- understand and use specialist vocabulary – plane geometry, mathematical analysis,integral and differential calculus (2 hours)
- analyse and understand specialist texts (2 hours)

TEACHING METHODS:

The course focuses on communication activities in functional and situational context. It encourages students to speak with fluency and develop the four skills of reading, writing, listening and speaking by means of group and pair work, discussion, presentation, oral and written exercises.

LEARNING OUTCOMES:

Achieving language skills and competence on level B2 of the Common European Framework of Reference for Languages.

Upon successful completion of the course, the students:

- can describe hypothetical situations with the use of adequate language structures
- use modal verbs to express probability and give advice
- use with understanding Passive Voice
- can prepare and deliver a presentation on a topic concerning a branch of mathematics
- are familiar with and can use specialist vocabulary from geometry and mathematical analysis
- can name types of angles and triangles
- can cooperate with members of a group, exchange information, and discuss problems

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Classes – exam: a condition for receiving a credit is a positive mark for the exam.

STUDENT WORKLOAD:

Contact time:

- classes – 30 hours
- consultation – 5 hours

Private study – 25 hours, students systematically prepare for the examination.

RECOMMENDED READING:

1. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Student's Book,* Oxford University Press 2007
2. C. Oxenden, V. Latham-Koenig, P. Seligson, *New English File Workbook,* Oxford University Press 2007
3. J. Pasternak-Winiarska, *English in Mathematics*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2006

OPTIONAL READING:

1. *FCE Use of English* by V. Evans
2. L. Szkutnik, *Materiały do czytania – Mathematics, Physics, Chemistry,* Wydawnictwa Szkolne i Pedagogiczne
3. Internet articles
4. R. Murphy *English Grammar in Use*.

GAME THEORY

Course code: 11.1-WK-MAT-SP-TG
Type of course: optional
Language of instruction: English/Polish
Director of studies: prof. dr. hab. Andrzej Nowak
Name of lecturer: prof. dr. hab. Andrzej Nowak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	IV	Exam	
Class	30	2		Grade	

COURSE AIM:

Knowledge of foundations of game theory, and significance of game theory in modern economics.

ENTRY REQUIREMENTS:

Mathematical analysis 1 i 2, linear algebra, probability theory

COURSE CONTENTS:

Lecture

I. Non-cooperative games in statistical concept:

1. Normal form games (1 hr)
2. Zero-sum games. Von Neumann minmax theorem. (3 hrs)
3. n-person games and Nash equilibrium. Connection between existence of Nash equilibria and fixed point theorems of continuous mappings. (6 hrs)
4. Non-cooperative games in economics: Bertrand competition and Cournot oligopoly. (2 hrs)
5. Nash bargaining model. (3 hrs)

II. Extensive form games (dynamic games):

1. Games with complete information. Kuhn existence theorem. (2 hrs)
2. Kuhn algorithm. (1 hr)
3. Incomplete information games. (2 hrs)

III. Cooperative games:

1. Examples: voting games, linear production games. (2 hrs)
2. Core of a cooperative game, Non-emptiness of the Core In some games. (2 hrs)
3. Shapley value, Banzhaf value (axioms and construction). (3 hrs)

IV. Incomplete information games:

1. Bayesian games. Auctions. (3 hrs)

Class

I. Static noncooperative games:

1. Solutions of zero-sum games. (3 hrs)
2. n-person games and Nash equilibria. Examples: Prisonier Dilemma. The best response map. (6 hrs)

3. Non-cooperative games in economics: examples of Bertrand competition and Cournot oligopoly. (2 hrs)
4. Nash bagaining model. Searching for solution. (3 hrs)

II. Extensive form games (dynamic games):

1. Complete information games. Application of Kuhn algorithm for construction of Nash equilibria. (2 hrs)
2. Examples of incomplete information games. (2 hrs)

III. Cooperative games:

1. Examples of voting games, linear - production games. (1 hr)
2. Core of cooperative games, examples. (2 hrs)
3. Shapley value, Banzhaf value (computing). (3 hrs)

IV. Game Theory with imperfect information:

1. Bayesian games. Auctions. Examples of games (3 hrs)

V. Colloquium: (4 hrs).

TEACHING METHODS:

Lectures and classes

LEARNING OUTCOMES:

Student

1. understands importance of mathematics for modelling conflicts and cooperations in economic models (K_W01, K_W03),
2. Knows basic minimax theorems and Nash equilibrium and their applications. (K_W04),
3. Understands idea of Nash bargaining model (K_W01),
4. Can construct and analyze game tree in simple problems. (K_U25, K_U26),
5. Knows basic models of cooperative games. (K_W01),
6. Knows Shapley and Banzhafs solution for cooperative games (K_W03),
7. Knows how to use simple stochastic models in game analysis (K_U32),

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Evaluation of individual exercises, final exam and grade.

STUDENT WORKLOAD:

Contact hours
Lecture – 30 hrs
Class – 30 hrs
Office hours – 15 hrs (5 hrs for lecture and 10 for class)
Total 75 hrs (3 ECTS)
Individual work
Preparing to lecture – 25 hrs
Preparing to class – 35 hrs
Preparing to exam – 40 hrs
Total: 100 hrs (4 ECTS)
Total hours and points per course: 175 hrs (7 ECTS)

RECOMMENDED READING:

1. Fudenberg, D. Game theory. MIT Press, Boston, 1991.
2. Owen, G. Teoria gier. PWN, Warszawa, 1975.
3. Osborne, M.J. A course in game theory. MIT Press, Boston, 1994.
4. Płatkowski, T. Wstęp do teorii gier. Uniwersytet Warszawski, Warszawa 2011.
5. Straffin, P.D. Teoria gier. Scholar, Warszawa, 2004.

OPTIONAL READING:

1. Myerson, R.B. Game theory: an analysis of conflict. Harvard University Press, 1997.
2. Owen, G. Game theory. EG Publishing, New York, 1995.

GENERAL ALGEBRA

Course code: 11.1-WK-MAT-SP-AO
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Joanna Skowronek-Kaziów
Name of lecturer: dr Joanna Skowronek-Kaziów

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					4
Lecture	30	2	III or V	Exam	
Class	30	2		Grade	

COURSE AIM:

In the end of this course the students know and understand the basic theorems concerning groups, rings, fields and lattices theory and they can applicate and use the notions and theorems from the abstract algebra in codes, cryptography and combinatorics.

ENTRY REQUIREMENTS:

Linear Algebra 1 and 2.

COURSE CONTENTS:

1. Prime numbers, The Basic Theorem of Arithmetics, congruences of integer numbers, Euler Totient function, Euler theorem. Definitions and properties of operations in the algebraic structures.
2. Groups, abelian groups, cyclic groups, subgroups, permutation groups, torsion and torsion-free groups. Cayley's theorem and Lagrange's Thorem for groups. Morphisms of groups, normal subgroups, simple groups, congruences in groups. Quotient groups, Isomorphism theorem for groups. Sylow's Thorem.
3. Rings, subrings, ideals, congruences in rings, quotient rings. Isomorphism theorem for rings, principal ideals, prime ideals, Maximal ideals. Chinese theorem. Fields, simple fields, finite fields.
4. Polynomial rings in one and many indeterminates, polynomial roots, symmetric polynomials. Bezout's theorem, Gauss's theorem, Eisenstein-Shönemann's criterion. Algebraic elements over a field, minimal polynomial. Extensions of fields. Fields algebraically closed. Hilbert's zeros Theorem.
5. Lattices, modular and distributive lattices, sublattices, examples. Dedekind-Birkhoff theorem. Boolean algebras.

TEACHING METHODS:

Traditional lectures; Solving appropriate selected exercises in the class.

LEARNING OUTCOMES:

K_W05++: The student knows examples of groups, rings, fields and lattices. Also can describe subalgebras of a given algebra using the appropriate theorems.

K_U17+: recognizes algebraic structures in different fields of mathematics (sets of numbers , matrices, functions, sequences, vectors and complex numbers with respect to the appropriate operations);

K_W04+: knows the basic theorems of General Algebra and their proofs – Lagrange Theorem for groups and Group Isomorphism Theorem .

K_U04+: applies Hasse diagrams to describe the lattices of normal subgroups of a group or of ideals of a ring.

K_U05++: can create the new objects by constructing the quotient algebras or cartesian products of algebras.

K_U08+: knows examples of algebraic and transcendental numbers.

K_K06+: can search information in the literature to prove some additional facts and theorems.

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Verifying the level of preparation of students and their activities during the classes. The student has to receive the positive grade from two tests with tasks of different difficulty which help to assess whether students have achieved effects of the course in a minimum degree (40% of the final grade). Written exam (60% of the final grade).

STUDENT WORKLOAD:

Contact hours

lecture – 30 hours

class – 30 hours

consultation – 3 hours (lecure) + 3 hours (class) =6 hours

Sum: 66 hours (2 ECTS)

Independent work

preparing to lecture – 17 hours

preparing to class – 17 hours

preparing to exam – 20 hours

Sum: 54 hours (2 ECTS)

Sum for the course: 120 hours (4 ECTS)

RECOMMENDED READING:

1. Białynicki-Birula, Zarys algebry, BM tom 63, PWN, Warszawa, 1987.
2. M. Bryński, Algebra dla studentów matematyki, PWN, Warszawa 1987.
3. Gleichgewicht, Algebra, Oficyna GiS, 2002.
4. W. J. GILBERT, W. K. NICHOLSON, MODERN ALGEBRA WITH APPLICATIONS, A JOHN WILEY & SONS, INC., PUBLICATION (http://cs.ioc.ee/~margo/aat/Gilbert%20W.J.,%20Nicholson%20W.K.%20Modern%20algebra%20with%20applications%20(2ed.,%20Wiley,%202004)(ISBN%200471414514)(347s).pdf)
5. J. Rutkowski, Algebra abstrakcyjna w zadaniach, PWN, Warszawa, 2000.

OPTIONAL READING:

1. G.Birkhoff,T.C.Bartee, Współczesna algebra stosowana, PWN,Warszawa,1983.
2. S. Burris, H. P. Sankappanavar , A Course in Universal Algebra, (http://orion.math.iastate.edu/cliff/BurrisSanka.pdf)
3. M. Bryński, J. Jurkiewicz, Zbiór zadań z algebry, PWN, Warszawa 1985.
4. A.I. Kostrykin, Wstęp do algebry, cz. I, III, PWN, Warszawa, 2005.
5. R. Lidl, Algebra dla przyrodników i inżynierów, PWN, Warszawa 1983.
6. A. Mostowski, M. Stark, Algebra wyższa, cz. I, II, III, PWN, 1966.

REMARKS:

The participation in this course is obligatory. Written exam.

GEOMETRY

Course code: 11.1-WK-MAT-SP-G
Type of course: compulsory
Language of instruction: Polish/English
Director of studies: dr hab. Krzysztof Przesławski, prof. UZ
Name of lecturer: dr hab. Krzysztof Przesławski, prof. UZ , dr Krystyna Białek, dr Andrzej Kisielewicz, dr Magdalena Łysakowskai

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					5
Lecture	30	2	III	Exam	
Class	30	2		Pass/Fail	

COURSE AIM:

The course has to main goals: developing skills of 'geometrizing' mathematical problems, solving geometric problems by algebraic methods.

ENTRY REQUIREMENTS:

Linear algebra 2

COURSE CONTENTS:

Wykład

Affine and Euclidean point spaces.

1. Affine combination of points; affine independency; examples of affine spaces; isomorphisms of affine spaces; a standard model of an affine space. Affine mappings. (4h)
2. Affine subspaces: hyperplane, line. Particular subsets of an affine space: line segments, convex sets, simplices. Sets of points in a general postion. Convex hull, polytope as a convex hull of a finite set. Caratheodory's theorem. Radon's theorem, Helly's theorem. (6h)
3. Euclidean point spaces: distance, ball, classification of isometries. (2h)
4. Halfspaces: geometric interpretation of linear inequations. Paralleotopes, cubes. (2h)
5. Closed convex sets; the distance of a point from a convex set and a hyperplane. (2 godz.)
6. Volume of a set – volume of a parallelotope and a simplex; Brunn–Minkowski inequality; John ellipsoid. (8h)

Projective spaces

1. Definition, basic properties, projective mappings(2h)

Quadric surfaces

1. Classification of conics and general quadrics.(4h)

Class

1. Exercises in elementary geometry (4h)
2. Elements of spherical geometry, spherical polytopes (formulas to be derived as exercises). Euler's formula for convex and spherical polytopes. Applications. (4h)
3. Compositions of isometries of the plane and the space. (3h)
4. Applications of Helly's theorem. (2h)
5. Finding the distance from a point to a set. (2h)
6. Finding the Minkowski's sum of convex figures and estimation of the volume of the sum – isoperimetric inequality. (2)
7. Minkowski's theorem on lattice points (the proof as a series of exercises); applications (2h)
8. Discussion over essays. (2h)
9. Informal introduction to the Euler characteristics– counting the Euler characteristic of selected set (e.g. closed surfaces). (2h)
10. Properties of conics and quadric surfaces (4h)
11. Class test (2h)

TEACHING METHODS:

Traditional lecturing, solving problems under the supervision of the instructor, preparing presentations or essays (collaborative effort).

LEARNING OUTCOMES:

1. Student is able to describe isometries of Euclidean (point) spaces. (KW04+)
2. (S)he knows the notions of a hiperplane, general affine subspace, halfspace. (S)he knows geometric interpretation of a linear system of equations (inequations). (S)he knows the notions of a a parallelotope and a simplex. (KU19+)
3. (S)he knows certain applications of convex sets. (S)he understands the notion of volume and its connexion with determinants. (S)he is capable to repeat the proof of the Minkowski theorem on lattice points. (KU18+, KW02+)
4. (S)he knows the formula for the area of a spherical polytope, and is able to use it in the proof of the Euler formula.
5. (S)he is able o find the Euler characteristic of selected geometric sets. (KU06+)
6. (S)he knows the notions of a conic and a (general) quadric, and is able to decide by analizing the equation of a conic (quadric) what kind of conic (quadric) it represents.
7. (S)he is capable to cooperate. (KK03+,KK06+) (in connexion with common work on essay or presentation)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Preparation of the students and their active participation is assessed during each class by their instructor.
2. Class test with problems of diverse difficulty helping to assess whether a student achieved minimal outcomes.
3. Essay or presentation prepared by a team.
4. Written examination: It consists of three theoretical problems (sth need to be proved or explained) and four practical problems (sth need to be calculated, checked or found).

Final grade = 0.4 x class grade + 0,6 x exam grade. In order to be allowed to take the exam a student has to have a positive class grade.

STUDENT WORKLOAD:

Contact hours
lecture – 30h
class – 30h
consultations – 5h for lectures + 5h for classes=10h
Subtotal: 70h (2.5 ECTS)
Homework
preparation for lectures – 20h
preparation for classes – 30h
preparation for the exam – 30h
Subtotal: 80h (2.5 ECTS)
Total for the whole course: 150 godz. (5 ECTS)

RECOMMENDED READING:

1. M. Berger, *Geometry I and II*, Universitext, Springer.

OPTIONAL READING:

1. H. Hopf, *Differential Geometry in the Large*, LNM 1000, Springer, 1989.
2. J. Matoušek, *Lectures on discrete geometry*, Springer, 2002.
3. M. Aigner, G. M. Ziegler, *Proofs from the BOOK*, Springer 2004.

INFORMATION TECHNOLOGY

Course code: 11.3-WK-IiE-SP-TI
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: mgr inż Andrzej Majczak
Name of lecturer: mgr inż Edward Ciaś
mgr inż Andrzej Majczak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Laboratory	30	2	I	Grade	

COURSE AIM:

Repetition and complete information of the basis of computer science: computer construction, operating systems, word processing, spreadsheets, presentation creating, basis of web pages design and Internet services.

ENTRY REQUIREMENTS:

Basis of computer skill knowledge at the secondary school scope.

COURSE CONTENTS:

Laboratory

1. Construction and the basics of computer.
2. Operating systems.
3. Word processing.
4. Spreadsheets.
5. Presentation creating.
6. Creating web pages and Internet services.
7. Basis of the JavaScript language.
8. Colloquium.

TEACHING METHODS:

Individual work at the computer. Processed material according to instructions that every student gets at the beginning of class. Discussions leading to deepen knowledge and understanding of the processed material.

LEARNING OUTCOMES:

Student:

1. Knows the basics of computer construction. (K_W08+)
2. Knows and understands the concept of an operating system, and can name operating system examples, know their types and application. (K_W08+)
3. Is able using text editors correctly format the text containing among others mathematical formulas, tables, charts. (K_U28+)

4. Knows at least two spreadsheets and their basic functionality. (K_W08+); (K_U28+)
5. Is able to develop their own presentation using programs to create presentations. (K_K05+)
6. Is able to create a simple web page, which contains interaction with user. (K_U26+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Checking the degree of student preparation and their activities during the classes.
2. Performing two tasks to assess.
3. Written colloquium at the end of the course.

STUDENT WORKLOAD:

Contact hours

laboratory – 30 hours

consultations – 2 hours

Together: 32 hours (1 ECTS)

Individual work

preparation for laboratory – 12 hours

preparation to colloquium – 8 hours

Together: 20 hours (1 ECTS)

Together: 52 hours (2 ECTS)

RECOMMENDED READING:

1. Aksoy P., Denardis L., Information technology in theory, Cengage Learning, 1 edition, 2007.
2. Czarny P., Komputer PC w biurze i nie tylko, Helion, 2008.
3. Danowski B., Tworzenie stron WWW w praktyce, Wydanie II, Helion, 2007.
4. Sokół M., Internet. Kurs, Wydanie III, Helion, 2011.
5. Sokół R., ABC Linux, Wydanie II, Helion, 2010.
6. Wrotek W., Informatyka Europejczyka. Technologia informacyjna, Helion, 2006.

OPTIONAL READING:

1. Diller A., LaTex. Wiersz po wierszu, Helion, 2001.
2. Gajda W., HTML, XHTML i CSS. Praktyczne projekty, Wydanie II, Helion, 2011.
3. Glass G., Ablem K., Linux dla programistów i użytkowników, Helion, 2007.
4. Howil W., Po prostu OpenOffice.ux.pl 3.x, Helion, 2010.Lampart L., LaTex. System opracowywania dokumentów, WNT, 2004.Mendrala D., Szeliga M., Swiatelski M., ABC systemu Windows XP PL, Wydanie II, Helion, 2006.
7. Nisan N., Schocken S., Elementy systemów komputerowych. Budowa nowoczesnego komputera od podstaw, WNT, 2008.Rychlicki-Kicior K., Podstawy obsługi komputera. Pierwsza pomoc, Wydanie II, Helion, 2011.
9. Silberschatz A., Galein P.B., Gagne G., Podstawy systemów operacyjnych, WNT, 2006.
10. Sokół M., Tworzenie stron WWW. Ćwiczenia praktyczne, Wydanie III, Helion, 2011.
11. Sokół M., OpenOffice.ux.pl 3.1. Ćwiczenia praktyczne, Helion, 2010.
12. Sokół M., Sokół R., XHTML, CSS i JavaScript. Pierwsza pomoc, Helion, 2009.

INTRODUCTION TO OPTIMIZATION

Course code: 11.1-WK-MAT-SP-PO
Type of course: optional
Language of instruction: English
Director of studies: prof. Andrzej Cegielski
Name of lecturer: prof. Andrzej Cegielski, dr. Robert Dylewski

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2		Examination	
Laboratory	30	2		Written test	

COURSE AIM:

The lecture should give a general knowledge on mathematical foundation of optimization, in particular on necessary and sufficient optimality conditions, on basic optimization methods and appropriate software.

ENTRY REQUIREMENTS:

Linear algebra 1 and 2, mathematical analysis 1 and 2.

COURSE CONTENTS:

1. **Backgrounds**
   Optimization problems and their classification. Various forms of optimization problems and the relationships among the problems. Elements of linear algebra, of differentiation and of convex analysis.
2. **Optimality conditions**
   Basic optimality conditions. Necessary and sufficient optimality conditions of the first order and of the second order for unconstrained minimization. Convex optimization problem. Duality
3. **Unconstrained minimization methods**
   Line search. General form of descent methods and their convergence. Methods: steepest descent, conjugate gradients, Newton, DFP and BFGS.

TEACHING METHODS:

Traditional lecture, laboratory with application of appropriate software

LEARNING OUTCOMES:

Student

- Can construct mathematical models for simple optimization problems (K_U25++)
- Knows and understands the graphic method for two-dimensional optimization problems (K_U11++)

- Knows and understands the properties of convex functions as well as necessary and sufficient optimality conditions (K_W04++, K_W07++)
- Is able to apply Kuhn-Tucker theorem and basic minimization methods to simple optimization problems (K_U12+++, K_U25+, K_U26++)
- Knows and applies an appropriate software to symbolic calculus and to simple optimization problems (K_W08++, K_W09, K_U25+, K_U26++)
- Understands the necessity of an application of mathematical methods in practical problems (K_K05+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Checking the activity of the student
2. Written tests
3. Checking the ability of application of an appropriate software
4. Written examination

The final grade consists of the lab's grade (50%) and the examination's grade (50%)

STUDENT WORKLOAD:

Contact hours
- Participation in lectures: 30h
- Participation in laboratory: 30h
- Consultations: 2h (lecture), 3h (laboratory)

Independent work
- Preparation for lectures: 25h
- Preparation for laboratory: 25h
- Preparation for exercises: 30h
- Preparation for examination: 25 h

Total: 170 h (6 ECTS)

RECOMMENDED READING:

1. M. Brdyś, A. Ruszczyński, Metody optymalizacji w zadaniach, WNT, Warszawa, 1985.
2. A. Cegielski, Podstawy optymalizacji, skrypt do wykładu
3. W. Findeisen, J. Szymanowski, A. Wierzbicki, Teoria i metody obliczeniowe optymalizacji, PWN, Warszawa, 1980.
4. Z. Galas, I. Nykowski (red.), Zbiór zadań z programowania matematycznego, część I, II, PWN, Warszawa, 1986, 1988.
5. W. Grabowski, Programowanie matematyczne, PWE, Warszawa, 1980.
6. J. Stadnicki, Teoria i praktyka rozwiązywania zadań optymalizacji, WNT, Warszawa, 2006.

OPTIONAL READING:

1. M. S. Bazaraa, H. D. Sherali, C. M. Shetty, Nonlinear Programming, Third Edition, J. Wiley&Sons, Hoboken, NJ, 2006
2. D. P. Bertsekas, Nonlinear Programming, Athena Scientific, Belmont, MA, 1995
3. J.E. Dennis, R.B. Schnabel, Numerical Methods for Unconstrained Optimization and Nonlinear Equations, SIAM, Philadelphia 1996.
4. R. Fletcher, Practical Methods of Optimization, Vol I, Vol. II, John Willey, Chichester, 1980, 1981.
5. C. Geiger and Ch. Kanzow, Numerische Verfahren zur Lösung unrestingierter Optimierungsaufgaben, Springer-Verlag, Berlin, 1999.
6. C. Geiger and Ch. Kanzow, Theorie und Numerik restringierter Optimierungsaufgaben, Springer-Verlag, Berlin, 2002.

INTRODUCTION TO THE MATHEMATICS OF FINANCE

Course code: 11.5-WK-MAT-SP-WMF
Type of course: optional
Language of instruction: English/Polish
Director of studies: dr hab. Longin Rybiński, prof. UZ
Name of lecturer: dr hab. Longin Rybiński, prof. UZ
dr Dorota Głazowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	III, V	Grade	
Laboratory	30	2		Grade	

COURSE AIM:

The student should accomplish basic tools for money time-value analysis, investment analysis, asset pricing and risk analysis, comparing and building investment strategies with derivatives.

ENTRY REQUIREMENTS:

Calculus 1, 2, Linear Algebra 1, Probability Theory

COURSE CONTENTS:

Lecture:

1. Simple, compound and continuous interest. Nominal and effective rates.
2. Mathematical models for varying rates.
3. Standard and nonstandard annuities and perpetuities.
4. Cash flows – present value, future value, internal rate of return, modified internal rate of return; investment cash flows.
5. Payment of a debt – schedule for a short term and long term debts; actual percentage rate.
6. Term structure of interest rates and yield curves. Bonds – zero-coupon bonds and coupon bonds; duration and convexity; immunization and matching assets and liabilities.
7. Pricing derivative securities – Black Scholes formula and Cox-Ross_Rubinstein formula.
8. Basics of portfolio theory; Capital Asset Pricing Model and Arbitrage Pricing Theory.
9. Von Neumann–Morgenstern expected utility.

Laboratory:

1. Present value and future value of payment in case of simple, discrete and continuously compound interest. Equivalence of nomianal and effective rate, equivalence of interest and discount rate.
2. Calculating present and future value of cash flow for constant and varying rates; annuities and perpetuities.

3. Internal rate of return (numerical aspects and spreadsheet calculation) and modified internal rate of return.
4. Tools for investment analysis: cash flow net present value, internal rate of return, profitability index, playback period. Solving practical problems.
5. Debt repayment plans. Calculation of payments and IRR based comparison of various debt repayment schedules.
6. Derivative securities (futur es, european and american and options) and basic option strategies – pricing in spreadsheet.

TEACHING METHODS:

Lectures – with conversation and online usage of financial and insurance data.

Laboratory – the use of spreadsheet functions, individual problem solving, individual project report.

LEARNING OUTCOMES:

Student knows how:

- interpret functional relationships, tables, formulas and aply mathematical models to practical problems,
- project and compare investment strategies, using the basic notions of financial mathematics and spreadsheets,
- explore relevant literature and databases.

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Assessment of written test, ongoing review of laboratory work, project assessment. The final grade is a weighted mean of lecture grade (60%) and laboratory grade (40%).

STUDENT WORKLOAD:

Lectures 30 h.

Individual literature study and web data exploration 30 h.

Preparation for written test 30 h.

Laboratory 30 h.

Unassisted preparation for laboratory work 15 h.

Project consulting 15 h.

The work on project and report 15 h.

Total: 165 h (6 ECTS)

RECOMMENDED READING:

1. M. Dobija, E. Smaga, Podstawy matematyki finansowej i ubezpieczeniowej, PWN, Warszawa, 1995.
2. E. Nowak (red.), Matematyka i statystyka finansowa, Fundacja Rozwoju Rach., Finanse, Warszawa, 1994.
3. M. Podgórska, J. Klimkowska, Matematyka finansowa, PWN, Warszawa, 2005.
4. Piasecki K., Modele matematyki finansowej, PWN, Warszawa, 2007.

OPTIONAL READING:

1. A. Weron, R. Weron, Inżynieria finansowa, WNT, Warszawa, 1998.
2. Capiński M., Zastawniak T., Mathematics for Finance, Springer, 2003.
3. P. Brandimarte, Numerical Methods in Finanace, John Wiley & Sons, New York, 2002.

LINEAR ALGEBRA 1

Course code: 11.1-WK-MAT-SP-AL1
Type of course: compulsory
Language of instruction: Polish/English
Director of studies: dr hab. Krzysztof Przesławski, prof. UZ
Name of lecturer: dr hab. Krzysztof Przesławski, prof. UZ,
dr Magdalena Łysakowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	45	3	I	Exam	
Class	45	3		Pass/Fail	

COURSE AIM:

To equip students with knowledge concerning basic algebraic structures such as fields, groups, vector spaces

ENTRY REQUIREMENTS:

Secondary school mathematics.

COURSE CONTENTS:

Lecture

Fields

1. Number fields. (2h)
2. Operations. Axiomatic definition of a field. (2h)
3. The field of rational functions (1h)
4. The field of residue classes modlo p; Fermat's little theorem (3h)
5. Isomorphisms of fields; automorphisms. The characteristic of a field. (2h)
6. Complex numbers: conjugation, modulus, polar form, geometric interpretation of addition and multiplication, De Moivre's theorem, roots of complex numbers. (4h)
7. Fundamental theorem of algebra. Algebraic and transcendental numbers. (basic information) (1h)
8. Noncommutative fields: quaternions. (only briefly; students have to expand their knowledge by self study)(1h)

Permutations

1. Definition of a group; examples. (1h)
2. Parity of a permutation; alternating groups. (2h)
3. Decomposition of a permutation into disjoint cycles; decomposition into transpositions. (1h)

Vector spaces

1. Definition of a vector space; examples. (1h)
2. Linear independence; subspaces and spanning sets; basis; the Steinitz exchange lemma; dimension.(2h)
3. Linear transformations; spaces of linear homomorphisms; isomorphisms; linear transformations between coordinate spaces and their matrices; matrix multiplication and composition of linear transformations; algebras over a field: algebras of linear endomorphisms. (3h)
4. Rank of a matrix; the kernel and image of a linear transformation. (3h)
5. Matrix of a linear transformation with respect to arbitrary bases. (2h)
6. Dual space; dual basis; double dual and the canonical isomorphism between a space and its double dual; transpose of a linear transformation; transposed matrix. (2h)

Determinants

1. Determinant of a square matrix; multilinearity of determinant. (2h)
2. Determinant of a product of two matrices; determinant of a linear endomorphism. (2h)
3. Laplace expansion; inverse of a matrix. (1h)
4. General linear group, special linear group; group of upper triangular matrices. (1h)

Systems of linear equations

1. Existence of solutions (2h)
2. Fundamental system of solutions; dimension of the space of solutions. (2h)
3. Form of the solution to the system Ax=b, when A is an invertible matrix. (1h)
4. Gauss-Jordan elimination. (1h)

Class

Fields

1. Rational and irrational numbers; examples. Number fields; examples. (3h)
2. Two-argument operations and their properties. (1h)
3. Modular computations: tables of operations, inverse elements; binomial coefficients (exercises with the use of mathematical induction); applications of Fermat's little theorem. (2h)
4. Complex numbers: finding products of numbers, and the inverse and the canonical form of a number. (2h)
5. Finding the argument and the modulus of a number. Roots. (2h)
6. Solving equations with complex coefficients. (2h)
7. Class test. (2h)

Permutations

1. Finding products of permutations. Inverses. Decompositions of permutations into cycles and transpositions. The sign of a permutation. (4h)

Vector spaces

1. Examples of vector spaces (2h)
2. Verification of linear independency; bases (2h)
3. Calculating values of linear mapping. Finding the kernel and the image of a linear mapping in some simple cases. (4h)
4. An algorithm for finding the rank of a matrix. (2h)
5. Class test. (2h)

Determinants

1. Applications of 2x2 determinants: the area of a parallelogram and a triangle. (2h)
2. 3X3 determinants: the volume of a parallelepiped. (2h)
3. Calculating certain determinants of large size. (4h)

Systems of linear equations

1. Finding the inverse of a matrix. (1h)
2. Checking the consistency of a linear system (2h)
3. Finding a fundamental system of solutions by Gauss-Jordan elimination. (2h)
4. Class test. (2h)

TEACHING METHODS:

Traditional lecturing, solving problems under the supervision of the instructor.

LEARNING OUTCOMES:

1. Student knows and understands the notion of a field, also knows basic examples of this structure. (KW05 +)
2. (S)he is capable to perform simple operations on elements of fields, as e.g.: finding the inverse of an element; evaluating a power of an element in the field of residues modulo p; evaluating a root of a complex number. (KU01+)
3. (S)he knows the proof of Fermat's little theorem. (KW04+)
4. (S)he is able to calculate the sign of a permutation. (KU18+)
5. (S)he uses the notion of a vector space, linear transformation, matrix, and is capable to solve simple problems formulated with the use of these notions. She/ he is able to find the matrix of a linear transformation with respect to given bases. (KW03+,KU16+, KU20+)
6. (S)he is able to check whether a given system of vectors is linearly independent, and to find the rank of a matrix. (KU19+)
7. Student knows the notion of determinant, and is able to calculate a determinant using the Laplace method. (S)he knows geometric interpretation of determinant in two and three dimensions . (KU18+)
8. (S)he is able to decide whether a given linear system is consistent. (S)he can solve linear systems of medium complexity. (KW03+, KU19+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Preparation of the students and their active participation is assessed during each class by their instructor.
2. Class tests with problems of diverse difficulty helping to assess whether a student achieved minimal outcomes.
3. Written examination: It consists of around 18 problems. Each problem consists of 2 or 3 statements. To solve a problem, one has only to decide whether the statements are true or false. For some of them, however, explanations are demanded.

Final grade = 0.4 x class grade + 0,6 x exam grade. In order to be allowed to take the exam a student has to have a positive class grade. In order to pass the exam a student has to have a positive exam grade.

STUDENT WORKLOAD:

Contact hours

lecture – 45h

class – 45h

consultations – 3h for lectures + 3h for classes=6h

Subtotal: 96h (3 ECTS)

Homework

preparation for lectures – 20h

preparation for classes – 40h

preparation for the exam – 24h

Subtotal: 84h (3 ECTS)

Total for the whole course: 180 godz. (6 ECTS)

RECOMMENDED READING:

1. Strang, Gilbert, *Linear Algebra and Its Applications*, Cengage Learning, 2005.

OPTIONAL READING:

1. G. Birkhoff, S. Mac Lane, *A Survey of Modern Algebra*, A.K. Peters, 1997.

LINEAR ALGEBRA 2

Course code: 11.1-WK-MAT-SP-AL2
Type of course: compulsory
Language of instruction: Polish/English
Director of studies: dr hab. Krzysztof Przesławski, prof. UZ
Name of lecturer: dr hab. Krzysztof Przesławski, prof. UZ, dr Magdalena Łysakowska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	II	Exam	
Class	30	2		Pass/Fail	

COURSE AIM:

The objective of the whole course (linear algebra 1 and 2) is to prepare participants to self-study of theoretical and practical problems involving methods of linear algebra. The aim of each student should be to master the material included in the recommended book.

ENTRY REQUIREMENTS:

Linear algebra 1

COURSE CONTENTS:

Lecture

Systems of linear equations

1. Characteristic equation; eigenvectors; eigenvalues; examples and applications. (4h)

Jordan decomposition

1. Algebraic sum of linear subspaces; direct sum. (1h)
2. Nilpotent endomorphisms; Jordan blocks. Invariant subspaces of an endomorphism. (2h)
3. Jordan decomposition of an endomorphism; Jordan normal form. (2h)

Euclidean spaces

1. Cosine theorem — geometric definition of a scalar product; scalar product in coordinate spaces. (1h)
2. Formal definition of a scalar product; norm; Schwarz inequality; angle between two vectors, triangle inequality; parallelogram law. (2h)
3. Orthogonality: Pythagorean theorem, orthonormal basis.(1h)
4. Gram–Schmidt algorithm, existence of an orthonormal basis, expansion of a vector with respect to an orthogonal basis, orthogonal complement. (3h)
5. Isomorphic Euclidean spaces; canonical isomorphism between a Euclidean space and its dual. (1h)
6. Conjugate of a linear transformation; spectral theorem for self-adjoint operations.

7. Orthogonal transformations; decomposition of a space into minimal invariant subspaces: rotations, reflections. Canonical matrix of an orthogonal transformation. Orientation.(5h)

Bilinear forms

1. Multilinear forms: skew forms, symmetric forms. (1h)
2. Bilinear symmetric forms: matrix of a form with respect to a given frame. (1h)
3. Diagonalization of a bilinear symmetric form; Sylvester's law. (2h)
4. Quadratic forms; polarization formula — the one-to-one correspondence between symmetric and quadratic forms. (1h)

Class

Systems of linear equations

1. Solving eigenvalue problems. (4h)

Jordan decomposition

1. Simple examples. Information on numerical packages. (2h)

Euclidean spaces

1. Finding the angle between vectors. Checking whether a given form is a scalar product (2h)
2. Finding an orthonormal basis by Gram–Schmidt orthogonalisation process. Gram's determinant and its geometrical interpretation. (5h)
3. Class test
4. Diagonalisation of simple self-adjoint transformations. (4h)
5. Classification of orthogonal transformations in dimensions 2 and 3. Composition of orthogonal transformations. Reduction of orthogonal matrices to their canonical forms – examples. (5h)

Bilinear forms

1. Matrix of a bilinear form. Decompsition of a form into skew and symmetric parts. (1h)
2. Diagonalization of bilinear forms (quadratic forms). (2h)

TEACHING METHODS:

Traditional lecturing, solving problems under the supervision of the instructor.

LEARNING OUTCOMES:

1. Student knows the notion of an eigenvalue, and an eigenvector. He is able to find them for problems of medium complexity (e.g. small size; presence of symmetries).(KU20+)
2. S(he) understands the meaning of an abstract Euclidean space for geometrisation of practical problems. (S)he is able to calculate an appropriate orthonormal basis. (S)he knows, and is able to find the Fourier expansion of a vector. (S)he can find the basis of eigenvectors for a simple self-adjoint transformation (KW07+ KU16+, KU21+).
3. (S)he is capable to reduce an orthogonal transformation to its canonical form in simple two or three dimensional cases(KU 21+).
4. (S)he knows how to reduce a quadratic form to its canonical form.
5. (S)he knows, on an operational level, basic theorems of linear algebra(KW03+,KW04++).

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

4. Preparation of the students and their active participation is assessed during each class by their instructor.
5. Class tests with problems of diverse difficulty helping to assess whether a student achieved minimal outcomes.
6. Written examination: It consists of around 18 problems. Each problem consists of 2 or 3 statements. To solve a problem, one has only to decide whether the statements are true or false. For some of them, however, explanations are demanded.

Final grade = 0.4 x class grade + 0,6 x exam grade. In order to be allowed to take the exam a student has to have a positive class grade. In order to pass the exam a student has to have a positive exam grade.

STUDENT WORKLOAD:

Contact hours
lecture – 30h
class – 30h
consultations – 5h for lectures + 5h for classes=10h
Subtotal: 70h (2.5 ECTS)
Homework
preparation for lectures – 40h
preparation for classes – 40h
preparation for the exam – 30h
Subtotal: 110h (3.5 ECTS)
Total for the whole course: 180 godz. (6 ECTS)

RECOMMENDED READING:

1. Strang, Gilbert, *Linear Algebra and Its Applications*, Cengage Learning, 2005.

OPTIONAL READING:

1. G. Birkhoff, S. Mac Lane, *A Survey of Modern Algebra*, A.K. Peters, 1997.

LOGIC AND SET THEORY

Course code: 11.1-WK-MAT-SP-LTM

Type of course: compulsory

Language of instruction: Polish/English

Director of studies: dr hab. Krzysztof Przesławski, prof. UZi

Name of lecturer: prof. dr hab. Marian Nowak,
dr hab. Krzysztof Przesławski, prof. UZ,
dr Dorota Głazowska, dr Agniesz Oelke

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	I	Exam	
Class	30	2		Pass/Fail	

COURSE AIM:

Familiarize students with structures underlying contemporary mathematics.

ENTRY REQUIREMENTS:

Secondary school mathematics

COURSE CONTENTS:

Lecture

1. **Propositional calculus**
   Logical connectives. Boolean valuations. Tautologies. Rules of inference. (3 godz.)
2. **Sets**
   Membership relation, set inclusion; equality of sets. Operations on sets: union, intersection, set difference, symmetric difference, complement. De Morgan laws. Cartesian product of two sets. (3h)
3. **Quantifiers**
   Definition, basic properties. Operations on arbitrary families of sets. (2h)
4. **Binary relations and functions**
   Domain and codomain. Frequently occurring relations. Function as a relation. Indexed families of sets: uions, intersections. Sentential functions – schema of specification, Russel's paradox. Injections, surjections, bijections; restriction and extension of a function; composition of a function; inverse functions. Images and counterimages. Methods of defining functions. (4h)
5. **Mathematical induction**
   Peano axioms . The principle o mathematical induction. Alternative formulations. Recurrent sequences. Counting finite sets; exclusion-inclusion principle. (3h)
6. **Relations (cont.)**
   Equivalence relations: Equivalence classes vs partitions. Quotients constructions: rational numbers. Generalized products; generalized relations. (3h)

7. **Cardinality of a set**
   Countable sets: the integers, the rational and algebraic numbers. Cantor–Bernstein theorem. Cantor theorem on power sets. Uncountability of the reals; other sets equinumerous with continuum: e.g. cardinality of a square. Continuum hypothesis. Cardinal numbers (brief information). (7h)
8. **Orders**
   Distinguished elements of partial orders: the greatest and the maximal elements, upper bound etc. Complete lattices – Tarski's fixed point theorem. Isomorphism of partially ordered sets: realisation of a partial order by inclusion. Preference relations. Dense and continuous linear orders. Well-ordered sets. (3h)
9. **Axiom of choice**
   Hausdorff's theorem on the existence of a maximal chain. Kuratowski-Zorn lemma; the existence of the Hamel bases. Well-ordering principle; cardinal numbers revisited. (2h)

Class

1. **Propositional calculus**
   Computing the logical value of a propositional expression for given values of its logical variables. Checking whether a given propositional expression is a tautology. Equivalent propositions – expressing a proposition in an equivalent form with the use of given connectives. (3h)
2. **Sets**
   Algebra of sets: checking whether two algebraic formulas involving sets and operations on sets represent the same set. Simple laws and their proofs. (3h)
3. **Quantifiers**
   Writing down theorems using quantifiers and logic symbols. (3h)
4. **Relations and functions**
   Checking properties of relations and functions. Finding domain and counterdomain of a function (relation). Compositions of functions. Manipulating with indexed families of sets. Images and counterimages. (3h)
5. **Class test (2h)**
6. **Mathematical induction**
   Examples of reasoning by induction. Functions defined inductively – finding their values and checking properties. (3h)
7. **Relations (cont.)**
   Verifying whether a given relation is an equivalence. Applications of equivalence relations to simple algebraic constructions.(3h)
8. **Cardinality**
   Comparing the cardinalities of two sets. Examples of countable and uncountable sets (Cantor's set). (5h)
9. **Orders**
   Checking properties of orders. Examples of dense subsets of the reals. Examples of complete lattices. Examples of well-ordered sets different from subsets of natural numbers. Transfinite induction (optionally). (3h)
10. **Class test (2h)**

TEACHING METHODS:

Traditional lecturing, solving problems under the supervision of the instructor.

LEARNING OUTCOMES:

1. Student understands the importance of mathematics for civilization.(KW01+)
2. (S)he understands the importance of reasoning in exact sciences. (S)he is able to illustrate this understanding by means of examples. (KW02+)
3. (S)he is able to use quantifiers and logical connectives. (KU02+)
4. (S)he is able to use simple diagrams in order to support reasoning (Venn and Hasse diagrams, graphs). (KU04+)
5. (S)he is able to perform simple inductive reasoning, and knows what is a recurrence sequence. (KU3+)
6. (S)he knows that there are infinite sets that are not equinumerous, and is able to justify it. (KU07+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Preparation of the students and their active participation is assessed during each class by their instructor.
2. Class tests with problems of diverse difficulty helping to assess whether a student achieved minimal outcomes.
3. Written examination: It consists of around 18 problems. Each problem consists of 2 or 3 statements. To solve a problem, one has only to decide whether the statements are true or false. For some of them, however, explanations are demanded.

Final grade = 0.4 x class grade + 0,6 x exam grade. In order to be allowed to take the exam a student has to have a positive class grade. In order to pass the exam a student has to have a positive exam grade.

STUDENT WORKLOAD:

Contact hours

lecture – 30h

class – 30h

consultations – 5h for lectures + 5h for classes=10h

Subtotal: 70h (2.5 ECTS)

Homework

preparation for lectures – 20h

preparation for classes – 40h

preparation for the exam – 30h

Subtotal: 90h (3.5 ECTS)

Total for the whole course: 160 godz. (6 ECTS)

RECOMMENDED READING:

1. K. Kuratowski, A.Mostowski, *Set theory*, North-Holland, 1976.

OPTIONAL READING:

1. M. Aigner, G. M. Ziegler, *Proofs from the BOOK*, Springer 2004.

REMARKS:

It is recommended to encourage students to use *Python* when solving exercises.

MATHEMATICAL ANALYSIS 1

Course code: 11.1-WK-MAT-SP-AM1
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: prof. dr hab. Witold Jarczyk
Name of lecturer: prof. dr hab. Witold Jarczyk,
prof. dr hab.Janusz Matkowski

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					10
Lecture	60	4	I	Exam	
Class	60	4		Grade	

COURSE AIM:

To acquaint students with basic notions of mathematical analysis: convergence of a sequence and series, limit, continuity and derivative of a function, also with connections between these notions.

ENTRY REQUIREMENTS:

To study the course it is necessary to be familiar with high school mathematics.

COURSE CONTENTS:

Lecture

I. Real numbers and complex numbers

1. Axioms of real numbers. Infimum and supremum (4 hours)
2. Root of a non-negative number (2 hours)
3. Complex numbers (4 hours)
4. Extended set of real numbers (1 hour)

II. Elementary functions I

1. Polynomials and rational functions. Power functions of a real variable, with rational exponent (1 hour)
2. Trigonometric functions of a real variable. Trigonometric form of a complex number (3 hours)

III. Sequences and series of numbers

1. Sequences of numbers and their convergence. Bounded sequences. Cauchy’s condition (2 hours)
2. Calculating limits of sequences (3 hours)
3. Upper limit and lower limit of a sequence (1 hour)
4. Fundamental properties of series of numbers (3 hours)
5. Series with non-negative terms. Comparison tests. Cauchy’s and d’Alembert criteria (4 hours)
6. Absolute and conditional convergence. Riemann’s theorem (2 hours)
7. Multiplying of series. Mertens’ theorem (2 hours)

IV. Limit and continuity of a function in a single variable

1. Limit of a function (2 hours)
2. Continuity. Intermediate value theorem (2 hours)
3. Global extrema. Extreme value theorem (1 hour)
4. Relationships of limits to continuity (1 hour)
5. Limits of functions of a real variable. One-sided limits (1 hour)
6. Limits of real-valued functions. Squeeze theorem (1 hour)
7. Asymptotes (1 hour)

V. Sequences and series of functions

1. Pointwise and uniform convergence (2 hours)
2. Series of functions. Weierstrass and Dirichlet tests (1 hour)
3. Power series. Cauchy-Hadamard's theorem (1 hour)

VI. Elementary functions II

1. Exponential functions. Logarithmic functions of a real variable (2 hours)
2. Power functions of a real variable (1 hour)
3. Trigonometric functions and inverse trigonometric functions (2 hours)

VII. Monotonic functions and convex functions

1. Monotonic functions (2 hours)
2. Convex functions (only a brief information; a part of the material, pointed out by the lecturer, should be prepared in student's own right basing on a literature indicated by the lecturer) (1 hour)

VIII. Elementary differential calculus I

1. Derivative and its interpretation. Differentiability of a function of a single real variable. Fundamental formulas concerning derivatives. Derivatives of elementary functions (2 hours)
2. Mean value theorems. Characterization of monotonicity (2 hours)
3. l'Hôspital's rule (1 hour)
4. Higher derivatives and Taylor formula (2 hours)

Class

I. Real numbers and complex numbers

1. Using axioms of real numbers in simple proofs (2 hours)
2. Learning basic properties sets of rational and irrational numbers. Determining infima and suprema of sets of real numbers (3 hours)
3. Drawing sets of complex numbers on the plane. Operations in complex numbers. Solving algebraic equations in complex domain (2 hours)

II. Elementary functions I

1. Examples of occurring elementary functions in simple problems outside mathematics (1 hour)
2. Finding the trigonometric form of a complex number. Determining roots of complex numbers (2 hours)

III. Sequences and series of numbers

1. Examining the convergence of sequences of numbers via definition (2 hours)
2. Examining the convergence of sequences of numbers by using Cauchy's condition (1 hour)
3. Examining the convergence of bounded monotonic sequences (2 hours)
4. Recurrent sequences. Making use of the squeeze theorem (1 hour)
5. Determining upper limits and lower limits (1 hour)
6. Examining the convergence of series of numbers. Using convergence tests (5 hours)
7. Calculating sums of series (1 hour)
8. Calculating Cauchy's product of series (1 hour)

Colloquium (2 hours)

IV.Limit and continuity of a function in a single variable

1. Examining the existence and determining the limit of a function (4 hours)
2. Checking the continuity of a function (2 hours)

V. Sequences and series of functions

1. Examining the uniform convergence of sequences of functions (2 hours)
2. Examining the uniform convergence of sequences of series of functions (2 hours)
3. Training of using Weierstrass' test to checking the uniform convergence of series of functions (1 hour)
4. Determining the center and radius of convergence of a power function (3 hours)

VI. Elementary functions II

1. Properties of exponential and trigonometric functions of a complex variable - training of elementary calculating proofs (2 hours)

Colloquium (2 hours)

VII. Monotonic functions and convex functions

1. Examining the convexity of functions via definition (1 hour)
2. Proving inequalities by examining the convexity of a suitable function (1 hour)

VIII. Elementary differential calculus I

1. Calculating derivatives via definition. Checking the differentiability. Finding tangent lines and normal lines to a curve (5 hours)
2. Making use of mean value theorems, verifying the monotonicity of differential functions, proving inequalities (3 hours)
3. Calculating limits of functions by using l'Hôspital's rule (2 hours)
4. Application of Taylor's formula to approximating of functions (2 hours)

Colloquium (2 hours)

TEACHING METHODS:

Traditional lecture; class where students, leaded by the teacher, solve exercises and discuss; team-work; work over a book; making use of internet

LEARNING OUTCOMES:

1. Student knows basic elementary functions (K_W04+,K_W07+)
2. He/she learns the notions of infimum and supremum, and examples illustrating them (K_W06+, K_W05+)
3. He/she wises and understands the notions of limit of a seuqence and a function and the notion of convergence of series (K_W07+)
4. Student knows what is a continuous function and learns basic properties of continuous functions (K_W07+, K_W04+)
5. He/she learns basic uniform convergence tests of series of functions (K_W04+)
6. Student knows the notion of power series and its basic properties (K_W07+, K_W04+)
7. He/she learns and understands the notion of a derivative and can prove Lagrange mean value theorem (K_W07+, K_W04+)
8. He/she wises what is l'Hôspital's rule (K_W07+,K_W04+)
9. He/she knows methods of mathematical analysis helpful while constructing models of medium complexity outside mathematics (K_W03+)
10. He/she can determine infima and suprema of sets of real numbers, knows examples of irrational numbers(K_U01+, K_U08+)
11. He/she is able to calculate limits of seuqences and functions (K_U10+)
12. He/she can examine the convergence of series of numbers and the uniform convergence of series of functions (K_U10+)
13. He/she examines the continuity of a function (K_U10+)
14. Student calculates derivatives and makes use of them while checking the monotonicity of a function (K_U12+)
15. He/she calculates limits of functiona via l'Hôspital's rule (K_U10+, K_U12+)
16. Student, on his/her own right, can search out information in a literature and internet (K_K06+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Verifying the extent of preparation of students and their activity during the classes.
2. Three colloquia with problems of various degree of difficulties, allowing to verify if students attained learning outcomes at the very least.
3. Exam (writ) with indicated point ranges.

The final grade is the arithmetic mean of those of the class and exam. A necessary condition to enter the exam is a positive grade of the classes. A necessary condition to pass the course is a positive grade of the exam.

STUDENT WORKLOAD:

Contact hours
lecture - 60 hours
classes - 60 hours
office hours - 20 hours (10 for the lecture and 10 for the classes)
total: 140 hours (5 ECTS)
Stand-alone work
preparation to lectures - 20 hours
preparation to classes - 60 hours
preparation to the colloquia - 20 hours
preparation to the exam - 40 hours
total: 140 hours (5 ECTS)
Total for the course: **280 hours (10 ECTS)**

RECOMMENDED READING:

1. Witold Jarczyk, Notatki do wykładu z analizy matematycznej, http://www.wmie.uz.zgora.pl/~`wjarczyk/materialy.html
2. Witold Jarczyk, Zadania z analizy matematycznej, http://www.wmie.uz.zgora.pl/~`wjarczyk/materialy.html
3. J. Douglas Faires, Barbara T. Faires, *Calculus,* Random House, New York.

OPTIONAL READING:

1. Józef Banaś, Stanisław Wędrychowicz, *Zbiór zadań z analizy matematycznej*, Wydawnictwo Naukowo-Techniczne, Warszawa, 1993.
2. Witold Kołodziej, *Analiza matematyczna*, Państwowe Wydawnictwo Naukowe, Warszawa, 1986.
3. Walter Rudin, *Podstawy analizy matematycznej*, Wydawnictwo Naukowe PWN, Warszawa, 2002.

MATHEMATICAL ANALYSIS 2

Course code: 11.1-WK-MAT-SP-AM2
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: prof. dr hab. Witold Jarczyk
Name of lecturer: prof. dr hab. Witold Jarczyk,
prof. dr hab.Janusz Matkowski

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					10
Lecture	60	4	II	Exam	
Class	60	4		Grade	

COURSE AIM:

To acquaint students with differential methods of examining extrema and the convexity of a function, with the notions of the primitive function and Riemann integral. The emphasis is placed on mastering calculating techniques, in particular those of integrating, and also on applications of differential and integral calculus. The next aim is to transfer basics of differential calculus on functions in several variables.

ENTRY REQUIREMENTS:

Mathematical Analysis 1. Logic and Set Theory. Linear Algebra 1.

COURSE CONTENTS:

Lecture

I. Elementary differential calculus II

1. Local extrema (1 hour)
2. Characterization of the convexity of a function (1 hour)
3. Relationships of the uniform convergence to differentiating (2 hours)
4. Differentiability of elementary functions (1 hour)
5. Primitive function (2 hours)
6. Algorithm of integrating rational functions (the material should be prepared in student's own right basing on a literature indicated by the lecturer)
7. Derivative of a function of a convex variable (a brief information) (1 hour)

II. Applications of differential calculus (the material should be elaborated in a written form by teams of students basing on a literature indicated by the lecturer)

1. Straight-line motion.
2. Applications to geometry.
3. Differential and approximate calculation.

4. Newton method.
5. Applications in economics.

III. Elementary integral calculus

1. Riemann integral and area. Basic properties of integral. Mean value theorem for integrals (8 hours)
2. Relationships of differentiation to integration. Newton-Leibniz fundamental theorem of calculus and its consequences (3 hours)
3. Relationships of uniform convergence to integration. Integrating series of functions (2 hours)
4. Improper integral (4 hours)

IV. Techniques of integration

1. Trigonometric substitutions (2 hours)
2. Euler's substitutions (2 hours)
3. Numerical integration: trapezoidal rule, Simpson's rule (the material should be prepared in student's own right basing on a literature indicated by the lecturer)

V. Applications of integral calculus

1. Exemplary applications of integration in geometry: areas of regions in the plane, volumes of solids, area of surfaces (2 hours)
2. Center of mass and moments. Theorems of Pappus (the material should be prepared in student's own right basing on a literature indicated by the lecturer)
3. Work and pressure (the material should be prepared in student's own right basing on a literature indicated by the lecturer)

VI. Polar coordinates and parametric equations

1. Polar coordinate system. Curves in polar coordinates. Area of a region bounded by a curve. Length of a curve (3 hours)
2. Parametric equations of a curve on the plane. Tangent line to a curve. Length of a curve (2 hours)

VII. Cartesian spaces

1. Scalars and vectors (1 hour)
2. Cylindrical coordinates and spherical coordinates (1 hour)

VIII. Functions of several variables

1. Level sets of functions of two or three variables (1 hour)
2. Limit and continuity (5 hours)

IX. Differential calculus of functions of several variables I

1. Directional and partial derivatives. Jacobi matrix and gradient (2 hours)
2. Differential and differentiability (7 hours)
3. Geometric interpretation of differentiability. Tangent plane and normal line (2 hours)
4. Regular mappings and diffeomorphisms (2 hours)
5. Implicit function theorem (3 hours)

Class

I. Elementary differential calculus II

1. Determination of local and global extrema. Proving inequalities by finding extrema. Function analysis (6 hours)
2. Examining the uniform convergence of sequences of functions and series of functions (2 hours)
3. Taylor's expansion of a function (4 hours)

III. Elementary integral calculus, **IV. Techniques of integration and V. Applications of integral calculus**

1. Calculating integrals by using definition (2 hours)
2. Integrating by parts and by substitution. Algorithm of integrating rational functions. Making use of Newton-Leibniz fundamental theorem of calculus (10 hours)

Colloquium (2 hours)

3. Convergence and integration. Integrating series of functions (2 hours)
4. Calculating areas of regions in the plane and volumes of solids (3 hours)
5. Determination of the center of mass and calculating the quantity of work (1 hour)

VI. Polar coordinates and parametric equations

1. Changing Cartesian coordinates into polar ones and conversely (2 hours)
2. Calculating areas of regions and length of curves described by polar equations (2 hours)
3. Determination of lines tangent to a curve described parametrically. Calculating areas of regions and length of curves described parametrically (3 hours)

VII. Cartesian spaces

1. Describing surfaces in spherical and cylindrical coordinates (1 hour)

Colloquium (2 hours)

VIII. Functions of several variables

1. Limits and continuity. Iterated limits. Continuity in separated variables (3 hours)

IX. Differential calculus of functions of several variables I

1. Finding directional derivatives, derivative and differential (5 hours)
2. Determination of tangent and normal lines and planes (2 hours)
3. Examining regularity and diffeomorphicity of mappings (3 hours)
4. Studying the problem of implicit functions (3 hours)

Colloquium (2 hours)

TEACHING METHODS:

Traditional lecture; class where students, leaded by the teacher, solve exercises and discuss; team-work completed with a written composition; work over a book; making use of internet.

LEARNING OUTCOMES:

1. Student knows necessary and sufficient conditions of the existence of local extrema of a differential function (K_W04+)
2. He/she knows simple examples of applications of differential calculus (K_W05+)
3. He/she learns the notion of Riemann integral and its interpretation (K_W07+)
4. He/she understands the proof of Newton-Leibniz fundamental theorem of calculus and is aware of consequences of that theorem (K_W04+,K_W07+)
5. He/she learns the algorithm of integrating rational functions (K_W07+)
6. Student knows basic methods of integration (K_W07+)
7. He/she realizes basic notions and results of differential calculus of functions in several variables (K_W07+, K_W04+)
8. He/she wises and understand implicit function theorem (K_W07+, K_W04+)
9. He/she knows methods of mathematical analysis helpful while constructing models of medium complexity outside mathematics (K_W03++)
10. He/she can make the function analysis (K_U12+)
11. He/she can give Taylor’s expansion of basic functions (K_U12+)
12. He/she makes use of various techiques of integration (K_U14+)
13. Student can apply integration to calculating areas of regions, volumes of solids, length of curves (K_U14+)
14. He/she can change the Cartesian coordinates into polar ones and vice versa (K_U11+)
15. He/she finds partial derivatives and differentials, determines tangent and normal lines and planes (K_U12+)
16. He/she can decide if a given mapping is a diffeomorphism (K_U12+)
17. Student is able single-handedly to seek out information in literature and internet (K_K06+)
18. He/she realizes the need of continued education (K_K01+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Verifying the extent of preparation of students and their activity during the classes.
2. Three colloquia with problems of various degree of difficulties, allowing to verify if students attained learning outcomes at the very least.
3. Written compositions elaborated a material indicated by the lecturer and prepared by teams of students.
4. Exam (writ) with indicated point ranges.

The final grade is the arithmetic mean of those of the class and exam. A necessary condition to enter the exam is a positive grade of the classes. A necessary condition to pass the course is a positive grade of the exam.

STUDENT WORKLOAD:

Contact hours

lecture - 60 hours

classes - 60 hours

office hours - 20 hours (10 for the lecture and 10 for the classes)

total: 140 hours (5 ECTS)

Stand-alone work

preparation to lectures - 20 hours

preparation to classes - 60 hours

preparation to the colloquia - 20 hours

preparation to the exam - 40 hours

total: 140 hours (5 ECTS)

Total for the course: **280 hours (10 ECTS)**

RECOMMENDED READING:

1. Witold Jarczyk, *Notatki do wykładu z analizy matematycznej*, http://www.wmie.uz.zgora.pl/~`wjarczyk/materialy.html
2. Witold Jarczyk, *Zadania z analizy matematycznej*, http://www.wmie.uz.zgora.pl/~`wjarczyk/materialy.html
3. J. Douglas Faires, Barbara T. Faires, *Calculus,* Random House, New York

OPTIONAL READING:

1. Józef Banaś, Stanisław Wędrychowicz, *Zbiór zadań z analizy matematycznej*, Wydawnictwo Naukowo-Techniczne, Warszawa, 1993.
2. Andrzej Birkholc, *Analiza matematyczna. Funkcje wielu zmiennych*, Wydawnictwo Naukowe PWN, Warszawa, 2002.
3. Witold Kołodziej, *Analiza matematyczna*, Państwowe Wydawnictwo Naukowe, Warszawa, 1986.
4. Walter Rudin, *Podstawy analizy matematycznej*, Wydawnictwo Naukowe PWN, Warszawa, 2002.

MATHEMATICAL ANALYSIS 3

Course code: 11.1-WK-MAT-SP-AM3
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: prof. dr hab. Witold Jarczyk
Name of lecturer: prof. dr hab. Witold Jarczyk, prof. dr hab.Janusz Matkowski

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					5
Lecture	30	2	III	Exam	
Class	30	2		Grade	

COURSE AIM:

To acquaint students with methods of examining extrema of functions in several variables, with integral calculus of multivariable functions, also with the notion of surface integral and basics of Fourier analysis.

ENTRY REQUIREMENTS:

Mathematical Analysis 1 and 2. Logic and Set Theory. Linear algebra 1 and 2.

COURSE CONTENTS:

Lecture

I. Differential calculus of functions of several variables II

1. Extrema (3 hours)
2. Extrema subject to a constrain (4 hours)

II. Integral calculus of multivariable functions

1. Double integrals. Iterated integrals. Double integrals in polar coordinates (4 hours)
2. Applications of double integrals: calculating areas of regions in the plane and surface areas in the space, center of mass and moments of inertia (2 hours)
3. Triple integrals and their applications. Convergence and integration (2 hours)

III. Line and surface integrals

1. Regular mappings and diffeomorphisms between spaces of different dimensions (5 hours)
2. Line integral and surface integral (3 hours)
3. Oriented line integral. Green's theorem (7 hours)

IV. Elements of Fourier analysis (the material should be elaborated in a written form by the student, basing on a literature indicated by the lecturer)

1. Trigonometric series.
2. Fourier series of a function. Criteria of the convergence of Fourier series.
3. Fejér's theorem.

CLASS

I. Differential calculus of functions of several variables II

1. Determination of local extrema of a function (4 hours)
2. Finding extrema subject to a constrain and global extrema (5 hours)

II. Integral calculus of multivariable functions

1. Calculating double integrals. Finding areas of regions (3 hours)
2. Calculating triple integrals. Finding volumes of solids (2 hours)

Colloquium (2 hours)

III. Line and surface integrals

1. Studying the regularity and diffeomorphicity of mappings between spaces of different dimensions. Parametrization of curves and surfaces (3 hours)
2. Calculating line integrals. Length of a curve (2 hours)
3. Calculating surface integrals (2 hours)
4. Calculating oriented line integrals (3 hours)
5. Applications of Green's formula. Calculating areas of regions (2 hours)

Colloquium (2 hours)

TEACHING METHODS:

Traditional lecture; class where students, leaded by the teacher, solve exercises and discuss; team-work; work over a book; making use of internet.

LEARNING OUTCOMES:

1. Student learns necessary and sufficient conditions of the existence of local extrema of functions in several variables and of extrema subject to a constrain (K_W07+, K_W04+)
2. He/she knows the notions of double and triple integrals (K_W07+)
3. He/she wises and understands basics of the theory of surface integrals and oriented surface integrals, and, among them, the proof of Green's theorem for rectangles (K_W07+, K_W04+)
4. Student is aware of Fourier series and criteria of their convergence (K_W04+)
5. He/she knows methods of mathematical analysis helpful while constructing models of medium complexity outside mathematics (K_W03+++)
6. He/she determines local extrema, extrema subject to a constrain, and global extrema of functions of several variables (K_U12+)
7. He/she calculates double and triple integrals and makes use of them while measuring areas of regions and volumes of solids (K_U14+)
8. He/she can calculate line and surface integrals (K_U13+, K_U14+)
9. Student can give the Fourier series of a function and investigate its convergence (K_U10+)
10. Student is able single-handedly to seek out information in literature and internet (K_K06+)
11. He/she can posed problems precisely (K_K02+)
12. He/she realizes the need of continued education (K_K01+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Verifying the extent of preparation of students and their activity during the classes.
2. Two colloquia with problems of various degree of difficulties, allowing to verify if students attained learning outcomes at the very least.
3. Exam (writ) with indicated point ranges.

The final grade is the arithmetic mean of those of the class and exam. A necessary condition to enter the exam is a positive grade of the classes. A necessary condition to pass the course is a positive grade of the exam.

STUDENT WORKLOAD:

Contact hours

lecture - 30 hours

classes - 30 hours

office hours - 10 hours (5 for the lecture and 5 for the classes)

total: 70 hours

Stand-alone work

preparation to lectures - 15 hours

preparation to classes - 35 hours

preparation to the colloquia - 15 hours

preparation to the exam - 15 hours

total: 80 hours

Total for the course: **150 hours (5 ECTS)**

RECOMMENDED READING:

1. Witold Jarczyk, *Notatki do wykładu z analizy matematycznej*, http://www.wmie.uz.zgora.pl/~`wjarczyk/materialy.html
2. Witold Jarczyk, *Zadania z analizy matematycznej*, http://www.wmie.uz.zgora.pl/~`wjarczyk/materialy.html
3. J. Douglas Faires, Barbara T. Faires, *Calculus,* Random House, New York

OPTIONAL READING:

1. Józef Banaś, Stanisław Wędrychowicz, *Zbiór zadań z analizy matematycznej*, Wydawnictwo Naukowo-Techniczne, Warszawa, 1993.
2. Andrzej Birkholz, *Analiza matematyczna. Funkcje wielu zmiennych*, Wydawnictwo Naukowe PWN, Warszawa, 2002
3. Witold Kołodziej, *Analiza matematyczna*, Państwowe Wydawnictwo Naukowe, Warszawa, 1986.
4. Walter Rudin, *Podstawy analizy matematycznej*, Wydawnictwo Naukowe PWN, Warszawa, 2002.

MATHEMATICAL SOFTWARE

Course code: 11.9-WK-MAT-SP-PM
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Tomasz Małolepszy
Name of lecturer: dr Tomasz Małolepszy

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					2
Laboratory	30	2	VI	Grade	

COURSE AIM:

The familiarization of the students with the capabilities of the mathematical software *Matlab*.

ENTRY REQUIREMENTS:

Computer Programming 1

COURSE CONTENTS:

1. First steps with Matlab.
   Overview of the basic capabilities of *Matlab*. Command window. Variables and types in Matlab. Basic constants. Auxiliary commands (*clc, clear, diary*). *Help* command. Standard mathematical functions in *Matlab*. (2 hours)
2. Vectors and matrices.
   Creating. Access to the elements. Removing elements. Basic functions operating on vectors and matrices. (3 hours)
3. String - char vectors.
   Creating. Basic functions operating on char vectors. Reading data - *input* function. Displaying text - *disp* function. *Sprintf* function - an advanced way to display the data. (4 hours)
4. Special types of arrays.
   Creating and operations on the following types of arrays: sparse matrices, cell and struct arrays. (4 hours)
5. Elements of the programming.
   Conditional statements - *if, switch*. Loops - *for, while*. Vectorization. M-files - scripts and functions. *Inline* functions. (4 hours)
6. Test. (2 hours)
7. Two- and three-dimensional graphics.
   Plot function (changing the type and the color of the graph). Labeling of axis and the graph, creating a legend. Creating graphs of the functions stored in files - *fplot* function. *Ezplot* function - parametric plots and graphs of implicit functions. *Matlab* functions plotting polygons and polylines. Basic operations on the graphic window (*figure*). Plotting curves in space (*plot3, ezplot3*). Plotting surfaces (*mesh, surf, ezsurf*). Animations. Import and export files. (4 hours)

8. Symbolic calculation in *Matlab*.

   Defining symbolic data - *sym* function. Solving equations and systems of equations - *solve* function. Computing limits (*limit*) and summing the series (*symsum*). Symbolic differentiation and integration (*diff* and *int* functions). Solving differential equations - *dsolve* function. (5 hours)

9. Test. (2 hours)

TEACHING METHODS:

To illustrate the capabilities of *Matlab*, during laboratory classes students will write computer programs solving some mathematical problems. In addition, in order for students to become more skilled at using *Matlab*, home exercises will be provided.

LEARNING OUTCOMES:

Student is able:

1. to use numerical software (Matlab) in two different ways: using built-in functions as well as writing own programs to solve some mathematical problems, (K_W08+, K_U15+, K_U27++, K_K01+)
2. to create and describe some simple 2D as well as 3D graphs, (K_U27++)
3. to perform symbolic calculations in Matlab. (K_W09+++)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Learning outcomes will be verified through two tests consisted of exercises of different degree of difficulty. A grade determined by the sum of points from these two tests is a basis of assessment.

STUDENT WORKLOAD:

Contact hours

Laboratories - 30 hours.

Laboratories' consultation hours - 5 hours.

Total - 35 hours (1 ECTS).

Individual work

Preparation to laboratories - 10 hours.

Homework exercises - 20 hours.

Total - 30 hours (1 ECTS).

Total time needed for this course: 65 hours (2 ECTS).

RECOMMENDED READING:

1. Jerzy Brzózka, Lech Dorobczyński, *Programowanie w Matlab*, Wydawnictwo Pracowni Komputerowej Jacka Skalmierskiego, Mikom, Warszawa 1998.
2. Wiesława Regel, *Wykresy i obiekty graficzne w programie Matlab*, Mikom, Warszawa 2003.
3. Desmond J. Higham, Nicholas J. Higham, *MATLAB guide*, SIAM, Philadelphia 2005.

OPTIONAL READING:

1. Anna Kamińska, Beata Pańczyk, *Ćwiczenia z ... Matlab. Przykłady i zadania*, Mikom, Warszawa 2002.

MODELLING IN FINANCE

Course code: 11.5-WK-MAT-SP-MF
Type of course: compulsory/optional
Language of instruction: English
Director of studies: dr hab. Mariusz Michta, prof. UZ
Name of lecturer: dr hab. Mariusz Michta, prof. UZ

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					7
Lecture	30	2	VI	Exam	
Laboratory	45	3		Grade	

COURSE AIM:

Knowledge of foundations of financial institutions, capital markets and methods of their products modeling

ENTRY REQUIREMENTS:

Probability theory , introduction to financial mathematics

COURSE CONTENTS:

Lectures:

1. Least squares method, linear financial time series and their applications In financial data analysis.
2. Risk modeling: different kinds of risks, financial risk, financial instruments, risk determinants, measures of risk, VAR, covariance methods, historical simulation method, simulation of scenario method.
3. Models for rate of return, volatility.
4. Blacka-Scholesa model and options pricing, historical and implied volatility.
5. Examples of egzotic options and their applications.
6. Modeling of demografic parameters: survival models, life time tables and parameters.
7. Models of insurance risk: principles of insurance premium calculations.
8. Models of insurances: risk process, reserves and Lundberg's model.
9. Ruin probabilisty.

Laboratory:

1. Simulations of continuous financial models.
2. Analysis of Real Word data using mathematical software packages.

TEACHING METHODS:

Lectures and computer laboratory analysis

LEARNING OUTCOMES:

1. Student understands civilization meaning of mathematics and its applications (K_W01).
2. Student knows bases of programming methods and understands their limitations (K_W08).
3. Student is able to interpret and explain function dependences, in the form of exemplars captivate, tables, diagrams, schemes and use it (them) in practical questions (K_U11)
4. Student is able to create investment strategies with utilization of financial instrument and value them yield and risk (K_U25)
5. Student is able to calculate premiums in life insurances and basic parameters of risk (K_U31,K_U33)
6. Student is able to formulate questions precisely, being as (serve) deep personal apprehension theme and recovery scanting element reasoning (K_K02).
7. Student understands requirement of popular introducing (presenting) mathematical methods in finance and insurance (K_K05).
8. Student is able to search out informations in literature independently also in foreign languages (K_K06).

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

Evaluation of individual exercises, final exam and grade

STUDENT WORKLOAD:

Lectures – 30 h
Laboratory-45h
Tutoring – 15 h (Lectures – 5 h.; Classes – 10 h)
Total: 90 h (3 ECTS)
Individual students` work
Preparing to lectures – 15 h
Preparing to classes– 55 h
Preparing to the exam – 30 h
Total: 100 h (4 ECTS)
Total hours and points per course 190 h (7 ECTS)

RECOMMENDED READING:

1. W. Ronka-Chmielowiec, Ryzyko w ubezpieczeniach-metody oceny, AE, Wrocław, 1997.
2. M. Dobija, E. Smaga, Podstawy matematyki finansowej i ubezpieczeniowej, WNT, Warszawa, 1996.
3. J. Hull, Kontrakty Terminowe i Opcje.Wprowadzenie, WIG-press, Warszawa, 1997.
4. J. Jakubowski, Modelowanie Rynk闌 Finansowych, Script, Warszawa, 2006.
5. E. Nowak (red.), Matematyka i statystyka finansowa, Fundacja Rozwoju Rach., Finanse, Warszawa, 1994.
6. P. Brandimarte, Numerical Methods in Finance and Econometrics. A MATLAB Based.
7. A.N. Shiryaev, Essentials of Stochastic Finance, Facts, Models, Theory, World Scientific, 1999.

OPTIONAL READING:

1. J. Jakubowski, A. Palczewski, M. Rutkowski, L. Stettner, Matematyka Finansowa, Instrumenty Pochodne, WNT, Warszawa, 2003.
2. Janicki, A Izydorczyk, Komputerowe Metody w Modelowaniu Stochastycznym, WNT, Warszawa, 2001.
3. M. Podgórska, J. Klimkowska, Matematyka Finansowa, PWN, 2005.

OPERATIONS RESEARCH 2

Course code: 11.1-WK-MAT-SP-BO2
Type of course: optional
Language of instruction: Polish/English
Director of studies: dr hab. Zbigniew Świtalski, prof. UZ
Name of lecturer: dr hab. Zbigniew Świtalski, prof. UZ, dr Robert Dylewski

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	V	Exam	
Laboratory	30	2		Grade	

COURSE AIM:

Knowledge of selected methods, models and applications of operations research.

ENTRY REQUIREMENTS:

Basic Linear Algebra, Discrete Mathematics, Probability Theory, Operations Research 1.

COURSE CONTENTS:

1. Mathematical modelling in operations research. Applications of operations research. (2 h.)
2. Selected models of discrete optimization and their applications. (6 h.)
3. Methods of solving the problems of discrete optimization. (2 h.)
4. Genetic algorithms. (2 h.)
5. Maximal flow problem. Ford-Fulkerson algorithm (2 h.)
6. Project scheduling methods. CPM method. (4 h.)
7. Travelling salesman problem. Little's algorithm. (4 h.)
8. Multicriteria programming. Interactive methods. (2 h.)
9. Dynamic programming. Decision trees. (2 h.)
10. Decision making under uncertainty. Stochastic programming. (4 h.)

TEACHING METHODS:

Lecture, laboratory classes

LEARNING OUTCOMES:

Student:

1. Knows basic models of discrete optimization. (K_W06)
2. Knows basic methods of solving the problems of discrete optimization, understands range and possibilities of their application. (K_W06)
3. Knows basic methods of multicriteria optimization (K_W06)

4. Is able to analyze a flow network and apply the F-F algorithm to finding maximal flow in a network. (K_U25, K_U29)
5. Is able to analyze a project network, to determine critical paths and slack times for nodes and activities. (K_U19, K_U20)
6. Is able to apply the Little's algorithm for solving the travelling salesman problem. (K_U19, K_U20)
7. Is able to apply basic decision rules under risk and uncertainty. (K_U19, K_U20)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Verification of activity of students during the laboratory classes.
2. Writing tests during the lab classes.
3. Writing exam.

Final score = Activity + writing tests (50 %), exam (50 %).

STUDENT WORKLOAD:

Contact hours:

Lecture – 30 h.

Laboratory – 30 h.

Consulting – 2 h. (lecture), 3 h. (lab.)

Self work:

Preparation for the lecture – 20 h.

Preparation for the lab. – 40 h.

Preparation for the exam – 35 h.

Total: **160 h**. (6 p. ECTS)

RECOMMENDED READING:

1. Cegielski, *Programowanie matematyczne - część 1 - Programowanie liniowe*, Uniwersytet Zielonogórski, Zielona Góra, 2002.
2. T. Trzaskalik, *Wprowadzenie do badań operacyjnych z komputerem*, PWE, Warszawa, 2003.
3. *Badania operacyjne* (red. W. Sikora), PWE, Warszawa, 2008.
4. F.S. Hiller, G.J. Lieberman, *Introduction to Operations Research,* McGraw-Hill, 2005.

OPTIONAL READING:

1. W. Grabowski, *Programowanie matematyczne*, PWE, Warszawa, 1982.
2. *Decyzje menedżerskie z Excelem* (red. T. Szapiro), PWE, Warszawa, 2000.
3. A.A. Korbut, J.J. Finkelsztejn, *Programowanie dyskretne*, PWN, Warszawa, 1974.

PROBABILITY THEORY

Course code: 11.1-WK-MAT-SP-RP
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: dr Marta Borowiecka-Olszewska
Name of lecturer: dr Marta Borowiecka-Olszewska

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					5
Lecture	30	2	III	Exam	
Class	30	2		Grade	

COURSE AIM:

Familiarizing students with the basic concepts, theorems and methods of reasoning related to the probability theory.

ENTRY REQUIREMENTS:

Getting a pass in Mathematical Analysis 1 and 2.

COURSE CONTENTS:

Lecture

1. Events and the probability
   - The revision of combinatorics. The classical definition of the probability. (2 hrs.)
   - The general definition of the probability. The definition and examples of the probability space and the event. Basic properties of the probability. (3 hrs.)
   - The geometrical probability. The conditional probability, the law of total probability and Bayes' rule. (3 hrs.)
   - The independence of events. The Bernoulli scheme, the most likely number of successes in the Bernoulli scheme. (2 hrs.)
2. Random variables and their distributions
   - The definition, examples and properties of the random variable. The distribution of the random variable. The cumulative distribution function of the random variable and its properties. The cumulative distribution function and types of distributions. (4 hrs.)
   - Absolutely continuous and discrete distributions. The probability density function and its property. Overview of the most important absolutely continuous and discrete distributions. Mixed distributions. The independence of random variables. (4 hrs.)
   - Multidimensional random variables. The joint and marginal distributions, multidimensional and marginal cumulative distribution functions, marginal probability density functions. Connections with independent random variables. Distributions of sums of independent random variables. (3 hrs.)

3. The expectation and moments of random variables
    - The expectation and moments of a random variable. Examples of basic absolutely continuous and discrete distributions. The expectation and moments of random variables of mixed distribution, basic properties and interpretations. The variance and the standard deviation of random variables, basic properties and interpretation. (4 hrs.)
    - The concept of the covariance and the correlation coefficient of random variables, their connections with independent random variables. Parameters of random vectors. The multidimensional normal distribution. (2 hrs.)
4. Limit theorems
    - Chebyshev's inequality, the weak and strong law of large numbers, the central limit theorem and their applications. (3 hrs.)

Class

1. Events and the probability
    - The binomial coefficient and its interpretation. The use of basic combinatorial schemes to exercises related to the classical definition of the probability. (4 hrs.)
    - Determination of elementary events and events. Basic properties of the probability. (2 hrs.)
    - Exercises that use the geometric probability, the conditional probability, the law of total probability and Bayes' rule. (2 hrs.)
    - Checking the independence of events. The calculation of probabilities of events with the assumption of independence. Exercises that use the Bernoulli scheme. (2 hrs.)
    - Colloquium (2 hrs.)
2. Random variables and their distributions, the expectation and moments of random variables
    - The verification whether some functions are random variables, cumulative distribution functions of some random variables. The determination of the cumulative distribution function of a random variable. The analysis of the distribution of a random variable on the basis of the cumulative distribution function. The verification whether some functions are probability density functions. The application of different types of discrete and absolutely continuous distributions in mathematical models. The application of normal distribution in exercises. (7 hrs.)
    - The determination of the joint and marginal distributions of two-dimensional random vectors using the tabular method. The determination of two-dimensional and marginal cumulative distribution functions, marginal probability density functions. The verification of the independence of random variables. Distributions of sums of independent random variables. (3 hrs.)
    - The determination of the expectation, moments and the variance of random variables. The properties of the expectation and the variance. The application in exercises. Calculations of the covariance and the correlation coefficient of random variables and their connections with the independence. The parameters of two-dimensional random vectors and two-dimensional normal distribution. (4 hrs.)
3. Limit theorems
    - The application of Chebyshev's inequality to estimate the probability of random variables. The application of the law of large numbers and the central limit theorem in exercises. (2 hrs.)
    - Colloquium (2 hrs.)

TEACHING METHODS:

A traditional lecture. Solving previously given tasks (exercises and short proofs) during the classes.

LEARNING OUTCOMES:

1. The student is able to explain concepts and give examples of the elementary event, the event, the probability measure, the probability space and the random variable. (K_W05+, K_U30++)
2. He is able to use the conditional probability, the law of total probability and Bayes' rule. He is able to check the independence of events and use the Bernoulli scheme. (K_U32+++, K_U29++)

3. He is able to analyse the distribution of random variables on the basis of e.g. a cumulative distribution function or a probability density function. He is able to apply different types of discrete and absolutely continuous distributions in mathematical models. (K_W03+, K_U31++)
4. He is able to calculate the probability of events, the expectation and the variance of random variables. He knows and is able to use limit theorems to estimate probabilities. (K_W04+, K_U33+++)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Checking the level of preparation of students and their activity during the classes.
2. Two colloquia with tasks of varying difficulty which allow to assess whether students have reached a minimum level of learning outcomes.
3. The exam in the form of a multiple-choice test, consisting of several dozen statements that require the verification on the basis of the acquired knowledge. The verification of statements is connected with the use of the theory or making simple calculations. The possible answers are Yes or No. The student may receive +1,-1 or 0 points for each statement.

To pass the class it is necessary to get passing scores in two colloquia. To take the exam it is necessary to pass the class. In order to pass the course it is necessary to get passing score in the exam. The final course grade is based on graded components: the class grade – 50% and the exam grade – 50%.

STUDENT WORKLOAD:

Contact hours

lecture – 30 hrs.

class – 30 hrs.

consultation – 5 hrs. for the lecture + 10 hrs. for the class = 15 hrs.

Total: 75 hrs.

Individual work

preparation for the lecture – 10 hrs.

preparation for the class – 30 hrs.

preparation for colloquia – 15 hrs.

preparation for the exam – 20 hrs.

Total: 75 hrs.

Total for the whole course: 150 hrs. (5 ECTS)

RECOMMENDED READING:

1. J. K. Misiewicz, Wykłady z rachunku prawdopodobieństwa z zadaniami, SCRIPT, Warszawa 2005.
2. J. Jakubowski, R. Sztencel, Wstęp do teorii prawdopodobieństwa, SCRIPT, Warszawa 2000.
3. T. Inglot, T. Ledwina, Z. Ławniczak, Materiały do ćwiczeń z rachunku prawdopodobieństwa i statystyki matematycznej, PWR, Wrocław 1984.
4. A. E. Plucińscy, Elementy probabilistyki, PWN, Warszawa 1982.
5. G. Grimmett, D. Welsh, Probability: an introduction, Oxford University Press, 1986.
6. G. Roussas, Introduction to probability, Elsevier Science, 2006.

OPTIONAL READING:

1. J. Jakubowski, R. Sztencel, Rachunek prawdopodobieństwa dla (prawie) każdego, SCRIPT, Warszawa 2002.
2. W. Krysicki, J. Bartos, W. Dyczka, K. Królikowska, M. Wasilewski, Rachunek prawdopodobieństwa i statystyka matematyczna w zadaniach, część I, PWN, Warszawa 1999.
3. A. Plucińska, E. Pluciński, Zadania z probabilistyki, PWN, Warszawa 1983.

TOPOLOGY

Course code: 11.1-WK-MAT-SP-T
Type of course: compulsory
Language of instruction: English/Polish
Director of studies: prof. dr hab. Marian Nowak
Name of lecturer: prof. dr hab. Marian Nowak

Form of instruction	Number of teaching hours per semester	Number of teaching hours per week	Semester	Form of receiving a credit for a course	Number of ECTS credits allocated
Full-time studies					6
Lecture	30	2	VI	Exam	
Class	30	2		Grade	

COURSE AIM:

Student should be familiar with the basic concepts of topology metric spaces: metric space, convergence in metric spaces, limit and continuity of mappings between metric spaces, separable, compact, complete and connected metric spaces.

ENTRY REQUIREMENTS:

Standard graduate courses in the set theory and mathematical analysis.

COURSE CONTENTS:

Lecture

Metric spaces

1. Elementary properties and examples of metric spaces. Function spaces.(2 hours)
2. The topology defined by a metric. Base of a metric space. System of neighborhoods. Interior and closure of sets. Open and closed sets.(2 hours)
3. Convergence of sequences in metric spaces. Comparison of metrics.(1hour)
4. Subspace of a metric space. Cartesian product of metric spaces.(2 hours)
5. Different sets in metric spaces.(1 hour)
6. Separable metric spaces - basic properties and examples.(1 hour)

Continuous mappings between metric spaces

1. Continuous mappings and their characterizations. Uniformly continuous mappings.(2 hours)
2. Homeomorphisms and isometries between metric spaces. Topological invariants.(1 hour)
3. Convergence of sequences of functions.

Complete metric spaces

1. Complete metric spaces. Elementary properties and examples.(2 hours)
2. Completion of metric spaces.(1 hour)
3. Baire category theorem. Baire category method.(1 hour)
4. Banach fixed point theorem.(1 hour)

Compact metric spaces

1. Compact metric spaces. Elementary properties and examples.(2 hours)
2. Characterizations of compact metric spaces. Borel-Lebesgue theorem.(2 hours)
3. Cartesian product of compact spaces.(1 hour)
4. Characterization of compact sets in Euclidean spaces.(1 hour)
5. Properties of continuous mappings on compact metric spaces. Weierstrass theorem.(3 hours)

Connected and arc connected metric spaces

1. Connected metric spaces. Elementary properties and examples.(1 hour)
2. Properties of continuous mappings on connected metric spaces.(1 hour)

Class

Metric spaces

1. Elementary properties of metrics. Euclidean spaces and function spaces.(2 hours)
2. Examining of metric conditions in concrete function spaces.(3 hours)
3. Comparison of metrics on the plane.(2 hours)
4. Examining of Cartesian products of metric spaces.(2 hours)
5. Operations on sets in metric spaces : calculation of the interior and the closure of sets in metric spaces.(4 hours)
6. Examining of convergence and of sequences in metric spaces.(2 hours)
7. Determination of different classes of sets in metric spaces.(2 hours)
8. Colloquium.(2 hours)

Continuous mappings

1. Examining of continuity and uniform continuity of functions on function spaces.(4 hours)
2. Examining of convergence of sequences in function spaces.(2 hours)

Topological properties basic classes of metric spaces

1. Examining of completeness of function metric spaces.(2 hours)
2. Characterization of compact and connected sets in metric spaces.(3 hours)
3. Colloquium.(2 hours)

TEACHING METHODS:

Traditional lecture, open to discussion; classes with lists of exercises and problems to be solved by students.

LEARNING OUTCOMES:

1. Student understands the concept of metric spaces and can indicate their elementary properties.(KW05++)
2. He/she is able to make basic topological operations on sets in a metric space , for example: the closure and the interior of sets.(KU23++)
3. Student knows the notions of convergence and the cluster point of a sequence in a metric and can determine characterizations of sequential convergence in basic classes of sequence and function metric spaces. He/she can compare different metrics defined on Euclidean spaces.(KU10++)
4. Student can construct new metric spaces by making use of a subspace of metric subspace and Cartesian products of metric spaces.(KU05+)
5. He/she can indicate basic properties of continuous mappings. He/she differentiate the concepts of continuous and uniformly continuous mappings. He/she can show examples of topological invariants.(KU09++)
6. Student knows basic classes of metric spaces: separable, compact, complete, connected metric spaces. He/she can show examples of such spaces.(KU23+)
7. Student knows elementary properties of functions defined on compact and connected metric spaces. He/she knows the proofs of the Weierstrass and Darboux theorems.
8. Student recognizes basic topological structures in objects considered in geometry and mathematical analysis.(KW05+)
9. Student can use topological notions in other fields of mathematics.(KU24+)

LEARNING OUTCOMES VERIFICATION AND ASSESSMENT CRITERIA:

1. Verification of preparation of students and their activity during classes.
2. Colloquia with various degree of difficulties, allowing to verify if students attained learning outcomes.
3. Exam (written) checks the understanding of the basic notions, examples and proofs of theorems basing on the indicated earlier examination criteria.

Passing the exam: the weighted mean of notes of the classes (40%) and the exam (60%).

A positive note of the classes is the necessary condition to be admitted to the exam.

A positive note of the exam attests the subject.

STUDENT WORKLOAD:

Contact hours

Lectures -30 hours.

Classes – 30 hours.

Office hours : 5 hours for lectures + 5 hours for classes

Jointly: 70 hours (3 ECTS)

Self-educational work

Preparation for the lecture - 40 hours

Preparation for the classes - 40 hours

Preparation for the exam - 30 hours

Jointly: 110 hours (3 ECTS)

Entire subject jointly: 180 hours (6 ECTS)

RECOMMENDED READING:

1. K. Janich, Topology, Springer-Verlag, New York Berlin Heidelberg Tokyo, 1984.
2. S. Gładysz, Wstęp do topologii, Wydawnictwo Naukowe PWN, Warszawa 1981.
3. W. Rzymowski, Przestrzenie metryczne w analizie, Wyd. UMSC, Lublin 2000.
4. J. Jędzejewski, W. Wilczyński, Przestrzenie metryczne w zadaniach , Wyd. UŁ. Łódz 2007.

OPTIONAL READING:

1. J. Jędrzejewski, Zarys teorii przestrzeni metrycznych, Wydawnictwo WSP Słupsk, 1999.
2. W. Archangielski, W.I. Ponomariow, Podstawy topologii ogólnej w zadaniach, PWN, Warszawa 1986.