P-Factor methods for nonregular inequality-constrained optimization problems

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Abstract

We are considering the nonregular optimization problem

$$\min_{x \in R^n} \phi(x)$$

$$\text{subject to } g_i(x) \leq 0,\ i = \overline{1,m} \tag{1}$$

where gradients $\nabla g_i(x^*)$ are linearly dependent at the solution x^*. Classical methods for solving such type optimization problems do not applicable since the Lagrange multiplier λ_0 in the equation

$$\lambda_0 \phi'(x^*) + \lambda_1 g_1'(x^*) + ... + \lambda_m g_m'(x^*) = 0,$$

may be equal zero.

We propose to reduce inequality-constrained optimization problem to equality- constrained optimization problem

$$\min_{x \in R^n} \phi(x)$$

$$\text{subject to } f_i(x,y) = g_i(x) + y_i^2 = 0, \qquad i = \overline{1,m} \tag{2}$$

Under assumptions of 2-regularity of the mapping

$$F(x,y) = (f_1(x,y), ..., f_m(x,y))^T$$

at the point (x^*, y^*)it followed that must be fulfilled equation

$$f'(x^*) + (F'(x^*,y^*) + P^{\perp} F''(x^*,y^*)h)\,\lambda^T = 0 \tag{3}$$

where

$\lambda^T = (\lambda_1, ..., \lambda_m)^T$, $h \in \mathrm{Ker} F'(x^*,y^*) \cap \mathrm{Ker}^{\,2} P^{\perp} F''(x^*,y^*)$ and P is orthoprojector onto $(\mathrm{Im} F'(x^*,y^*))^{\perp}$,and we can apply Newton method.

Keywords

p-regularity, singularity, factor-operator.

References:

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