Second-order conditions in vector optimization

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Abstract

The aim of this lecture is to present the second-order necessary and sufficient conditions for vector optimization problems. Our results generalize some of previously obtained results ([BP1, BP2, BZ, GGR]) concerning both scalar and vector optimization problems.

Main result: Let $f : \mathbb{R}^m \mapsto \mathbb{R}^n$ be a function minimized with respect to the pointed closed convex cone C with $\text{int}C \neq \emptyset$, which is continuous near $x \in \mathbb{R}^m$ and $\ell-$stable at x.

Assume that $\triangle_l(x) = \{\xi \in C' \cap S_{\mathbb{R}^n}; f^\ell(x;h)(\xi) = 0,\ \forall h \in S_{\mathbb{R}^m}\} \neq \emptyset$, and suppose that for each $h \in \mathbb{R}^m$ one of the following two conditions is satisfied:

(i) $\text{Limsup}_{t\downarrow 0} \frac{f(x+th)-f(x)}{t} \bigcap (-C) \neq \emptyset$,

(ii) $\underline{f}''^{\ell}_P(x;h)(\triangle_l(x)) > 0$.

Then x is isolated mininizer of second-order for f.

For $x \in \mathbb{R}^m$, $h \in S_{\mathbb{R}^m}$, $\xi \in C'$, we define

$$f^\ell(x;h)(\xi) = \liminf_{t\downarrow 0} \frac{\langle \xi, f(x+th) - f(x)\rangle}{t},$$

$$\underline{f}''^{\ell}_P(x;h)(\triangle_l(x)) = \liminf_{t\downarrow 0} \sup_{a \in \triangle_l(x)} \frac{\langle a, f(x+th) - f(x)\rangle - tf^\ell(x;h)(a)}{t^2/2}.$$

Further, Limsup denotes the Kuratowski upper limit set, and by $\ell-$*stability* at x, we mean that for some neighbourhood $U \subset X$ and $K > 0$ it holds

$$|f^\ell(y;h)(\xi) - f^\ell(x;h)(\xi)| \leq K\|y-x\|, \quad \forall y \in U,\ \forall h \in S_{\mathbb{R}^m},\ \forall \xi \in C' \cap S_{\mathbb{R}^n}.$$

Keywords: Second-order optimality conditions, Vector optimization, Isolated minimizer of second-order, Peano derivative.

References

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