Using the Peano derivative in unconstrained optimization

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Abstract

The main purpose of this lecture is to provide the second-order nonsmooth sufficient optimality condition for which the previous corresponding results ([1], [2], [3], [4]) can be obtained as a special cases. Our main result will be : **Theorem.** Let $f : \mathbb{R}^N \to \mathbb{R}$ be continuous near $x \in \mathbb{R}^N$ and let f be ℓ-stable at x. If $f^\ell(x;h) = \lim_{t\downarrow 0}[f(x+th) - f(x)]/t = 0$ for every $h \in S_{\mathbb{R}^N}$, and

$$\liminf_{t\downarrow 0} \frac{f(x+th) - f(x) - tf^\ell(x;h)}{t^2/2} > 0, \qquad \forall h \in S_{\mathbb{R}^N},$$

then x is an isolated minimizer of order 2 for f.

By ℓ-stability at x we mean that for some neighbourhood U of x and some $K > 0$ it holds:

$$|f^\ell(y;h) - f^\ell(x;h)| \le K\|y - x\|, \qquad \forall y \in U, \quad \forall h \in S_{\mathbb{R}^N}.$$

Keywords: locally Lipschitz function, regular function, $C^{1,1}$ function, Peano derivative, stable function, isolated minimizer of order k, Dini derivative.

References

[1] D. Bednařík, K. Pastor, Elimination of strict convergence in optimization, *SIAM J. Control Optim.*,43(3) (2004), pp.1063–1077.

[2] A. Ben-Tal, J. Zove, Directional derivatives in nonsmooth optimization, *J. Optim. Theory Appl.*,47(1985), pp.483–490.

[3] R. Cominetti, R. Correa, A generalized second-order derivative in nonsmooth optimization, *SIAM J. Control Optim.*,28 (1990), pp.789–809.

[4] I. Ginchev, A. Guerraggio, M. Rocca, From scalar to vector optimization , *Appl. Math.*, in print.