Sharp solutions to vector equilibria with applications to stability

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In this talk we discuss vector equilibrium problems (VEP) which encompass a large spectrum of problems, including vector optimization problems and vector variational inequalities.

Let X, Y be normed vector spaces. Let $\mathcal{K} \subset Y$ be a closed convex pointed cone in Y which induces the ordering relation: $x \leq y \Leftrightarrow y - x \in \mathcal{K}$ (with respective natural meanings for $\geq$, $\not\leq$, $\not\geq$).

Consider a mapping (bifunction) $F : C \times C \rightrightarrows Y$ defined on a subset C of X and such that

$$F(x,x) \geq 0 \quad \text{for} \quad x \in C.$$

Our goal is to investigate vector equilibrium problems of the form:

- *vector equilibrium problems* (VEP): find $x \in C$ such that

$$F(x,y) \not\leq 0 \quad \text{for all} \quad y \in C, \quad F(x,y) \not\geq 0 \tag{1}$$

  i.e.,

$$-F(x,y) \notin \mathcal{K} \quad \text{for all} \quad y \in C, \quad F(x,y) \not\geq 0.$$

To study Hőlder type continuities (pseudo-Hőlderness, Hőlder calmness) of solutions to parametric vector equilibria $(VEP)_u$ of finding all $x \in C(u)$ such that

$$F(u,x,y) \not\leq 0 \quad y \in C(u), \quad F(u,x,y) \not\geq 0,$$

we introduce several variants of strong pseudomonotonicity of vector-valued bifunctions. When applied to vector optimization problems, i.e. when $F(x,y) = \phi(y) - \phi(x)$ and $\phi : X \to Y$ is a mapping from X into Y the strong pseudomonotonicity reduces to sharpness of solutions. This latter notion has been succesfully applied to stability of paramteric vector optimization problems [3].

We say that F is *strongly pseudomonotone* ([4]) if there exists $\beta > 0$ such that for any pair of distinct points $x, y \in C$ we have

$$F(x,y) \not\leq 0 \to F(y,x) \notin \beta \|y - x\|^2 B_Y - \mathcal{K},$$

where B_Y denote the open unit ball in Y. Strong pseudomonotonicity strenghen pseudomonotonicity which appears in the existence results for vector equilibria (see e.g. [1]). For real-valued bifunctions F (with $Y = R$), the strong pseudomonotonicity has been used to study Hőlder stability of solutions to parametric (scalar) set-valued variational inequalitites (VI) ([7]), and (scalar) equilibrium problems ([2]). Hőlder continuities of solutions to parametric variational inequalities were also investigated in [2, 5, 7, 8] under strong monotonicity assumption.

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