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Discretization of quadratic optimal control problems governed by second order differential equations

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Abstract

We consider a class of control constrained quadratic optimal control problems governed by second order differential equations. For this class of control problems we study discretizations by finite element and by finite difference methods. Error estimates for the discrete controls are derived and some numerical results are presented.

Keywords: quadratic optimal control, finite element discretization, finite difference discretization.

Sharp solutions to vector equilibria with applications to stability

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Abstract

In this talk we discuss vector equilibrium problems (VEP) which encompass a large spectrum of problems, including vector optimization problems and vector variational inequalities.

Let X, Y be normed vector spaces. Let $\mathcal{K} \subset Y$ be a closed convex pointed cone in Y which induces the ordering relation: $x \leq y \Leftrightarrow y - x \in \mathcal{K}$ (with respective natural meanings for $\geq$, $\not\leq$, $\not\geq$).

Consider a mapping (bifunction) $F : C \times C \rightrightarrows Y$ defined on a subset C of X and such that

$$F(x,x) \geq 0 \quad \text{for} \quad x \in C.$$

Our goal is to investigate vector equilibrium problems of the form:

- *vector equilibrium problems* (VEP): find $x \in C$ such that

$$F(x,y) \not\leq 0 \quad \text{for all} \quad y \in C, \quad F(x,y) \not\geq 0 \tag{1}$$

i.e.,

$$-F(x,y) \notin \mathcal{K} \quad \text{for all} \quad y \in C, \quad F(x,y) \not\geq 0.$$

To study Hőlder type continuities (pseudo-Hőlderness, Hőlder calmness) of solutions to parametric vector equilibria $(VEP)_u$ of finding all $x \in C(u)$ such that

$$F(u,x,y) \not\leq 0 \quad y \in C(u), \quad F(u,x,y) \not\geq 0,$$

we introduce several variants of strong pseudomonotonicity of vector-valued bifunctions. When applied to vector optimization problems, i.e. when $F(x,y) = \phi(y) - \phi(x)$ and $\phi : X \to Y$ is a mapping from X into Y the strong pseudomonotonicity reduces to sharpness of solutions. This latter notion has been succesfully applied to stability of paramteric vector optimization problems [3].

We say that F is *strongly pseudomonotone* ([4]) if there exists $\beta > 0$ such that for any pair of distinct points $x, y \in C$ we have

$$F(x,y) \not\leq 0 \rightarrow F(y,x) \notin \beta\|y-x\|^2 B_Y - \mathcal{K},$$

where B_Y denote the open unit ball in Y. Strong pseudomonotonicity strenghen pseudomonotonicity which appears in the existence results for vector equilibria (see e.g. [1]). For real-valued bifunctions F (with $Y = R$), the strong pseudomonotonicity has been used to study Hőlder stability of solutions to parametric (scalar) set-valued variational inequalitites (VI) ([7]), and (scalar) equilibrium problems ([2]). Hőlder continuities of solutions to parametric variational inequalities were also investigated in [2, 5, 7, 8] under strong monotonicity assumption.

References

[1] M. Bianchi, N. Hadijsavvas and S. Schaible, *Vector Equilibrium Problems with generalized monotone bifunctions*, JOTA Vol. 92, No. 2, 527–542.

[2] M. Bianchi, R. Pini, *A note on stability for parametric equilibrium problems*, Operations Research Letters, Vol. 31 (2003), 445–450.

[3] E.M. Bednarczuk, *Weak sharp efficiency and growth condition for vector-valued functions with applications, optimization* 53 (2004), 455–474.

[4] E.M. Bednarczuk, Strong pseudomonotonicity, sharp efficiency and stability for parametric vector equilibria, to appear in the SMAI Journal.

[5] S. Dafermos, *Sensitivity analysis in variational inequalities*, Math. OR 13 (1988), 421–435.

[6] F. Giannessi, *Theorems of the Alternative, Quadratic Programs and Complementarity Problems*, in: Variational Inequalitites and Complementarity Problems, edited by Cottle R.W., Giannessi F., Lions J.L., John Wiley and Sons, New York 1980, 151–186.

[7] G. Kassay and J. Kolumban, *Multivalued parametric variational inequalities with α-pseudomonotone maps*, JOTA 107 (2000), 35–50.

[8] N.D. Yen, *Hőlder continuity of solutions to a parametric variational inequality*, Appl. Math, Optim, Vol. 31 (1995), 245–255

Using the Peano derivative in unconstrained optimization

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Abstract

The main purpose of this lecture is to provide the second-order nonsmooth sufficient optimality condition for which the previous corresponding results ([1], [2], [3], [4]) can be obtained as a special cases. Our main result will be:

Theorem. Let $f : \mathbb{R}^N \to \mathbb{R}$ be continuous near $x \in \mathbb{R}^N$ and let f be ℓ-stable at x. If $f^\ell(x;h) = \lim_{t\downarrow 0}[f(x+th) - f(x)]/t = 0$ for every $h \in S_{\mathbb{R}^N}$, and

$$\liminf_{t\downarrow 0} \frac{f(x+th) - f(x) - tf^\ell(x;h)}{t^2/2} > 0, \qquad \forall h \in S_{\mathbb{R}^N},$$

then x is an isolated minimizer of order 2 for f.

By ℓ-stability at x we mean that for some neighbourhood U of x and some $K > 0$ it holds:

$$|f^\ell(y;h) - f^\ell(x;h)| \leq K\|y - x\|, \qquad \forall y \in U, \quad \forall h \in S_{\mathbb{R}^N}.$$

Keywords: locally Lipschitz function, regular function, $C^{1,1}$ function, Peano derivative, stable function, isolated minimizer of order k, Dini derivative.

References

[1] D. Bednařík and K. Pastor, *Elimination of strict convergence in optimization*, SIAM J. Control Optim. 43 (3) (2004), 1063–1077.

[2] A. Ben-Tal and J. Zove, *Directional derivatives in nonsmooth optimization*, J. Optim. Theory Appl. 47 (1985), 483–490.

[3] R. Cominetti and R. Correa, *A generalized second-order derivative in nonsmooth optimization*, SIAM J. Control Optim. 28 (1990), 789–809.

[4] I. Ginchev, A. Guerraggio and M. Rocca, From scalar to vector optimization, Appl. Math., in print.

A modification of the alternating projection method

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Abstract

Let $A, B \subset \mathbb{R}^n$ be two closed, convex subsets. We deal with so called alternating projection method (APM)

$$x_{k+1} = P_A P_B x_k$$

which was introduced by von Neuman and studied by Deutsch, Dykstra and by Bauschke and Borwein. It is known that x_k converges to a fixed-point of the operator $T = P_A P_B$. We modify the APM in such a way that the Fejér monotonicity with respect to $\operatorname{Fix} P_A P_B$ and the convergence of x_k is preserved. Furthermore, we apply our results to the linear split feasibility problem (LSFP):

$$\text{find } x \in C \text{ such that } A^\top x \leq b,$$

where $C \subset \mathbb{R}^n$ is closed and convex, A is an $n \times m$ real matrix and $b \in \mathbb{R}^m$.

Adjoint methods for fast optimal control with inequality constraints

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Abstract

We present solution approaches to optimal control of large scale ordinary differential (ODE) and differential algebraic equation (DAE) models. The approaches are based on the direct multiple shooting method which transcribes the optimal control problem into a large but structured nonlinear program (NLP). This NLP is solved by a novel adjoint based Sequential Quadratic Programming (SQP) technique that avoids an expensive evaluation of constraint jacobians in each SQP iteration. Instead, it uses inexact jacobians that can be provided in various ways e.g. by evaluating them only once or by quasi Newton update formulae. In each SQP iteration an exact lagrange gradient is needed which can efficiently be computed by adjoint differentiation techniques within the underlying ODE/DAE solver. Convergence of the novel SQP algorithm in the presence of inequalities is shown under mild conditions, and the method is applied to optimal control problems that involve spatially discretized partial differential equations. Finally, we discuss how the ideas can be transferred to real-time optimal control algorithms, and present their application to feedback control of large scale processes from chemical engineering.

An iterative method for solving linear feasibility problem

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Abstract

Some optimization problems can be reduced to the solution of a system of linear inequalities (SLI). One of methods of solving the problem SLI is the surrogate constraints method, where the linear constraints are multiplied by some nonnegative weights and then added. The metric projection of the current approximation onto such a surrogate constraint is evaluated. We deal with a method, which is a modification of the surrogate constraints method. The first iteration is identical in both methods. In next iterations, for a given approximation $\bar{x}$, beside the violated constraints in $\bar{x}$, we take also into consideration the surrogate inequality, which we have obtained in the previous iteration.

The motivation for this research comes from the work of H. Scolnik *et al.* [1], who studied some projection methods to a system of linear equations.

Keywords: linear feasibility problem, iterative method, surrogate constraints, metric projection.

References

[1] H. Scolnik, N. Echebest, M.Y. Guardarucci and M.C. Vacchino, *A class of optimized row projection methods for solving large nonsymertric linear systems*, Applied Numerical Mathematics 41 (2002), 499–513.

[2] K. Yang and K.G. Murty, *New Iterative Methods for linear Inequalities*, Journal of Optimization Theory and Applications 72 (1992), 163–185.

Residual selection model for convex optimization problems

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Abstract

We present residual selection method for convex optimization problems, in particular for convex feasibility problems. We show that the residual selection method is a special case of the surrogate constraints method. We also present the results of numerical tests.

Keywords: convex optimization, convex feasibility, residual selection, surrogate constraints method.

References

[1] A. Cegielski and R. Dylewski, *Residual selection in a projection method for covex minimization problems*, Optimization 52 (2003), 211–220.

Differentiability of multifunctions and optimal control problems for differential inclusions

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Abstract

The main purpose of the talk is to extend the classical notion of Fréchet differentiability to multifunctions mapping points of a finite-dimensional normed space to compact convex subsets of another finite-dimensional normed space. We begin with introducing a notion of affinity for multifunctions.

Let X and Y be finite-dimensional normed spaces, $\mathcal{K}(Y)$ the collection of convex compact sets in Y.

A multifunction $\mathcal{A} : X \to \mathcal{K}(Y)$ is called *affine* if $\alpha\mathcal{A}(x_1)+(1-\alpha)\mathcal{A}(x_2) = \mathcal{A}(\alpha x_1 + (1-\alpha)x_2)$ for all $x_1, x_2 \in \mathrm{dom}\mathcal{A}$ and $\alpha \in [0,1]$.

As a dual counterpart to each affine multifunction $\mathcal{A} : X \to \mathcal{K}(Y)$ we introduce the conjugate mapping $\mathcal{A}^* : Y^* \to X^*$ and prove that $\mathcal{A}^*$ is a single-valued difference-sublinear mapping from Y^* into X^*. Moreover, we prove that for each difference-sublinear mapping $\mathcal{P} : Y^* \to X^*$ there exists an affine multifunction $\mathcal{A} : X \to \mathcal{K}(Y)$ such that $\mathcal{A}^* = \mathcal{P}$.

We demonstrate that affine multifunctions are completely characterized with their single-valued affine selections. We introduce the notion of an exposed selection as well as the notion of an extreme selection and prove that each affine multifunction can be presented as the closed convex hull of its exposed affine selections and as the convex hull of its extreme affine selections. These results extends the Straszewicz theorem and the finite-dimensional version of the Krein-Milman theorem from classisal convex analysis to affine multifunctions.

Using affine multifunctions as local (differential) approximations and the Hausdorff distance for defining a tangency relation between multifunctions, we extend the notion of Fréchet differentiability to multifunctions. We characterize Fréchet differentiability of multifunctions through the differential properties of their support functions.

In the concluding part of the talk we discuss applications above results to optimal control problems for systems described by differential inclusions.

Keywords: multifunction, Fréchet differentiability, Hausdorff metric, affine multifunction, difference-sublinear mappings, optimal control, differential inclusions.

Properties of path-following penalty methods applied to elliptic control problems

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Abstract

Considered are elliptic control problems of the following type

$$J(y,u) := \frac{1}{2}\int_\Omega (y(x;u)-z(x))^2 \;+\; \frac{\alpha}{2}\int_\Omega u^2(x) \to \ \text{min!}$$
$$\text{s.t.} \qquad -\Delta y = u \ \text{ in } \Omega, \quad y=0 \ \text{ on } \partial\Omega, \qquad u \in U_{ad} \tag{1}$$

with a bounded domain $\Omega \subset \mathbb{R}^n$, $\alpha > 0$, $z,a,b \in L_2(\Omega)$ and $U_{ad} = \{u \in L_2(\Omega) \,:\, a \le u \le b \text{ a.e. in } \Omega\}$. Finite element discretizations of the state as well as of the controls applied to (1) lead to finite dimensional optimization. General penalty-barrier path-following Newton methods (cf. [1]) for including the control constrains are studied and its convergence behavior in dependence of the discretization is analyzed. Further the application of the proposed path-following method in case of additional state constraints of obstacle type is discussed. As alternative way to handle the considered simple control constraints the projection approach of [2] that discretizes only the states is included into the study.

Keywords: path-following penalty methods, optimal control, elliptic state equations.

References

[1] C. Grossmann and M. Zadlo, *A general class of penalty/barrier path-following Newton methods for nonlinear programming*, Optimization 54 (2005), 161–190.

[2] M. Hinze, *A variational discretization concept in control constrained optimization: The linear-quadratic case*, Comput. Optim. Appl. 30 (2005), 45–61.

[3] U. Prüfert, F. Tröltzsch and M. Weiser, The convergence of an interior point method for an elliptic control problem with mixed control-state constraints, ZIB-Report 04-47, November 2004.

[4] S.J. Wright, Primal-Dual Interior Point Methods, SIAM Publ., Philadelphia 1997.

A decomposition method for variational inequality

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Abstract

Variational inequalities defined over a Cartesian product of sets are widely used in optimization: e.g. for saddle point problems, traffic equilibrium, nash equilibrium problems, etc. Moreover, any variation inequality can be formulated by Moreau–Yosida regularization as a problem with the decomposition property. The proximal point method with an additive regularization function allows us to reduce the original problem to a set of problems, which can be solved independently of each other.

Here, we present a decomposition algorithm. The convergence result admits an inexact solution of the subproblems and the approximation of the multi-valued operator by ϵ-enlargement. Some applications will be presented.

Keywords: variational inequalities, proximal point algorithm, decomposition, partial Dunn property, ϵ-enlargement.

Relations between generalizations of optimal control problems

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Abstract

Generalizations (or relaxations) of optimal control problems are often useful in order to examine the existence of a solution as well as to construct approximate solutions. But sometimes there may occur gaps between the original problem and its generalization. In this connection we present three known generalizations and deduce the relations between them.

Keywords: optimal control, generalization, relaxation.

References

[1] L. Bittner, Zur Existenz von Optimallösungen für das Problem des Wiedereintritts in die Atmosphäre und das Problem des Einschwenkens in eine Umlaufbahn, SFB 255, Report 20, München und Greifswald 1995.

[2] L. Bittner, Zur Existenz einer optimalen Lösung des Brachistochroneproblems mit Tangentialwinkelbeschränkungen, SFB 255, Report 29, München und Greifswald, 1995.

[3] S. Butzek, Aspekte des Zusammenhangs von Optimalsteuerproblemen und zuge-hörigen relaxierten Problemen, Dissertation, Greifswald 1997.

[4] S. Butzek and W.H. Schmidt, Relaxation Gaps in Optimal Control Processes with State Constaints, Conference Proceedings. ISNM 124, pp. 21–30, Birkhäuser 1998.

[5] R.V. Gamkredlidze, Principles of Optimal Control Theory, Plenum Press, überarbeitete Übersetzung, New York, London, 1978, russisches Original 1975.

[6] H. Irrgang, Eine Erweiterung der Rubio-Verallgemeinerung von Optimalsteuerproblemen mit Anwendung der Ky-Fan-Theorie über Systeme linearer Ungleichungen Dissertation, Greifswald 2001.

[7] T. Roubíček, Relaxation in Optimization Theory and Variational Calculus, Walter de Gruyter, Berlin, New York 1992.

[8] J.E. Rubio, Control and Optimisation, Manchester University Press, Manchester 1986.

[9] L.C. Young, Lectures on the Calculus of Variations and Optimal Control Theory, Chelsea Publishing Company, New York 1980.

A hybrid proximal projection algorithm for solving variational inequalities – numerical considerations

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Abstract

Standard algorithms for solving ill-posed variational inequalities have a bad performance while regularization methods behave stable. The classical inexact proximal point algorithm for finding zeros of a maximal monotone operator is a well-known regularization method which converges weakly to a solution [1]. In [2], Solodov and Svaiter introduced a hybrid proximal point algorithm which converges strongly under mild assumptions. Their main idea is to use a particular form of inexact solutions of the subproblems and to perform projection steps on the intersection of two halfspaces containing the solution set.

In this talk we consider different ideas for the numerical realization of this hybrid algorithm and show that to determine an inexact solution with the desired properties is a crucial step requiring special strategies. Furthermore, a numerical comparison with Rockafellar's classical inexact proximal point algorithm is presented. In particular the comparison will highlight some interesting facts about the monotonicity properties of the iterates.

Keywords: variational inequality, proximal point method, projection method, strong convergence.

References

[1] R.T. Rockafellar, *Monotone operators and the proximal point algorithm*, SIAM Journal on Control and Optimization 14 (1976), 877–898.

[2] M.V. Solodov and B.F. Svaiter, *Forcing strong convergence of proximal point iterations in a Hilbert space*, Mathematical Programming A87 (2000), 189–202.

[3] H. Schramm and J. Zowe, *A version of the bundle idea for minimizing a nonsmooth function: conceptual idea, convergence analysis, numerical results*, SIAM Journal on Optimization 2 (1992), 121–152.

$\boldsymbol{P}$-factor method for solving degenerate unconditional optimization problem

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Abstract

This work is concerned with irregular nonlinear unconstrained optimization problems:

$$\min_{x \in R^n} \varphi(x)$$

where $\varphi''(x^*)$ is singular at the solution point x^*. In this case methods do not applicable. We propose the 3-factor method for solving such type problems:

$$x_{k+1} = x_k - (\varphi''(x_k) + P_1\varphi'''(x_k)h + P_2\varphi^{(IV)}(x_k)h^2)^{-1} \cdot$$

$$\cdot(\varphi'(x_k) + P_1\varphi''(x_k)h + P_2\varphi^{(IV)}(x_k)h^2)$$

where P_1, P_2 – some orthoprojectors (see reference [1]) and

$$h \in Ker\varphi''(x^*) \cap Ker^2 P_1\varphi''(x^*).$$

Keywords: optimization, P-factor.

References

[1] A.A. Tret'yakov and J.E. Marsden, Factor-analysis of non-linear mappings, Communications on Pure and Applied Analysis, Vol. 2, No. 4, December 2003.

[2] V.G. Karmanov, Mathematical programming, Moscow 1986.

Bundle methods with approximate subgradient linearizations

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Abstract

We discuss proximal bundle methods for minimizing $f(u)$ subject to $h(u) \leq 0$, $u \in C$, where f, h and $C \subset \mathbb{R}^m$ are convex. We only require evaluating f, h and their subgradients with an accuracy $\epsilon \geq 0$, which is fixed but possibly unknown. The methods employ an exact penalty function with an updated coefficient, or a combination of the classic method of centers' improvement function with an exact penalty function, without needing a feasible starting point. They asymptotically find points with at least ϵ-optimal f-values that are ϵ-feasible. When applied to the solution of LP programs arising in column generation approaches to integer programming problems, they allow for ϵ-accurate solutions of column generation subproblems.

Our talk is partly based on joint research with Claude Lemaréchal, INRIA, France.

Keywords: nondifferentiable optimization, convex programming, proximal bundle methods, approximate subgradients, Lagrangian relaxation, column generation.

References

[1] K.C. Kiwiel, Methods of Descent for Nondifferentiable Optimization, Lecture Notes in Mathematics 1133, Springer-Verlag, Berlin 1985.

[2] K.C. Kiwiel, A proximal bundle method with approximate subgradient linearizations, SIAM J. Optim. (2005), in press.

Efficient calculation of sensitivities for optimization problems

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Abstract

Sensitivity information are required by numerous optimization algorithms to minimize a given problem. They can be computed within working accuracy using the forward mode of automatic differentiation (AD).

ADOL-C is an AD-tool for programs written in C or C++. Originally, tapes for values, operations and locations are written while evaluating the function. Subsequently, these tapes are used to compute the derivatives, sparsity patterns etc. By applying the recently implemented tapeless forward mode for scalar and vector calculations one completely avoids the generation of the tapes. Therefore, the computation of function and derivative values is performed directly from main memory which leads to smaller run times compared to the original approach.

Advantages and disadvantages of the tapeless forward mode will be discussed. Furthermore, run time comparisons for the two variants of the forward mode will be presented. They are based on some optimization problems that require the computation of sensitivity information.

Keywords: automatic differentiation, sensitivities, forward mode.

References

[1] A. Griewank, A. Kowarz, J. Utke, O. Vogel and A. Walther, *Documentation of ADOL-C*, part of the ADOL-C package.

Fuzzy goal programming – a new conception of deviations

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Abstract

There is proposed an approach to the goal programming problems in which the goals are given in the form of fuzzy numbers. It consists in a new concept of deviations for the imprecise (non-crisp) target values. The auxiliary problems permitting to obtain the solution of the initial goal-programming problem is formulated. The problem can be solved by means of existing non-linear programming methods. A real optimization problem is solved as an example.

Keywords: goal programming, fuzzy number, deviations.

Theory and application of nonlinear semidefinite programming

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Abstract

We consider static output feedback (SOF) control design problems and focus the discussion on SOF problems if the control system is described by partial differential equations. The discretization of those problems leads to large-scale nonlinear semidefinite programs (NSDPs). We discuss some theoretical and practical difficulties which arise in the solution of such problems. Finally, we consider some algorithmic strategies for solving the non-convex NSDPs and demonstrate the behavior of the solver on *COMPl$_e$ib* benchmark examples.

Modeling, analysis and optimal control of systems described by hemivariational inequalities

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Abstract

In the paper we present a survey on the mathematical modeling of nonconvex and nonsmooth problems arising in the mathematical theory of contact mechanics which is a growing field in engineering and scientific computing. The approach to such problems is based on the notion of hemivariational inequality and our presentation focuses on three aspects. First we present the ideas leading to inequality problems encountered in mechanics and we formulate different forms of hemivariational inequalities. Then we give results on the existence and uniqueness of solutions to hemivariational inequalities of elliptic, parabolic and hyperbolic types. For these classes of hemivariational inequalities we formulate optimal control problems and provide conditions under which they admit optimal solutions. Finally we indicate the mechanical problems in viscoelasticity, thermoviscoelasticity, heat conduction and the fluid flow problems to which our results can be applied.

Keywords: hemivariational inequality, subdifferential, multivalued, nonsmooth, control problem, nonconvex, contact, friction, viscoelasticity.

References

[1] Z. Denkowski and S. Migórski, *A system of evolution hemivariational inequalities modeling thermoviscoelastic frictional contact*, Nonlinear Anal. 60 (2005), 1415–1441.

[2] Z. Denkowski, S. Migórski and N.S. Papageorgiou, An Introduction to Nonlinear Analysis: Theory, Kluwer Academic/Plenum Publishers, Boston, Dordrecht, London, New York 2003.

[3] Z. Denkowski, S. Migórski and N.S. Papageorgiou, An Introduction to Nonlinear Analysis: Applications, Kluwer Academic/Plenum Publishers, Boston, Dordrecht, London, New York 2003.

[4] S. Migórski, Dynamic hemivariational inequality modeling viscoelastic contact problem with normal damped response and friction, Appl. Anal. 2005, in press.

[5] S. Migórski, Boundary hemivariational inequalities of hyperbolic type and applications, J. Global Optim. 2005, in press.

[6] S. Migórski and A. Ochal, *Boundary hemivariational inequality of parabolic type*, Nonlinear Anal. 57 (2004), 579–596.

[7] S. Migórski and A. Ochal, *Hemivariational inequalities for stationary Navier-Stokes equations*, J. Math. Anal. Appl. 306 (2005), 197–217.

[8] S. Migórski and A. Ochal, *Hemivariational inequality for viscoelastic contact problem with slip dependent friction*, Nonlinear Anal. 61 (2005), 135–161.

[9] Z. Naniewicz and P.D. Panagiotopoulos, Mathematical Theory of Hemivariational Inequalities and Applications, Marcel Dekker, Inc., New York, Basel, Hong Kong 1995.

[10] P.D. Panagiotopoulos, Hemivariational Inequalities, Applications in Mechanics and Engineering, Springer-Verlag, Berlin 1993.

Sufficient optimality conditions for multidimensional control problems

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Abstract

In this paper, we develop a dual approach to the dynamic programming for the optimal control problems (subject to parabolic, elliptic and wave equations) in a multidimensional case. The idea of our method consists in defining, instead of the value function, a new function which satisfies a dual first-order partial differential equation of dynamic programming. We then prove a suitable verification theorem and introduce the concept of a dual feedback control. The sufficient optimality conditions thus obtained are analogous to their one-dimensional counterparts.

Keywords: dual dynamic programming, dual feedback control, optimal control problems, sufficient optimality conditions.

On some dynamic bilateral frictional contact problem in viscoelasticity*

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Abstract

We present the mathematical model of bilateral frictional contact between a viscoelastic body and a rigid foundation. The body is permanently in contact with the foundation and there is no gap. Such settings can be found in many engineering applications where the relative motion of machine parts is such that contact must be maintained at all times; for example the contact between the piston rings and the engine block in a car. In our model, the dynamic equilibrium equation is considered with the viscoelastic constitutive relationship of the Kelvin-Voigt type. We assume the normal component of the displacement on the contact surface vanishes and we profit from the Coulomb law of friction. Such boundary conditions lead to the coupling of the contact conditions in the tangential and in the normal directions. The problem is modelled by an evolution hyperbolic variational inequality.

We establish the existence result for the dynamic bilateral frictional contact problem in viscoelasticity. The proof consists of two main steps. First we study the corresponding variational inequality when the stress field on the contact boundary is supposed to be known. The existence and uniqueness of this auxiliary problem is obtained by employing the surjectivity result for multivalued L-pseudomonotone and coercive operators. Then we use the Banach fixed point theorem and obtain existence and uniqueness result for our problem.

Keywords: bilateral frictional contact problem, variational inequality, viscoelasticity.

References

[1] M. Shillor, M. Sofonea and J.J. Telega, Models and Analysis of Quasistatic Contact. Variational Methods, Springer 2004.

[2] S. Migórski and A. Ochal, *Hemivariational inequality for viscoelastic contact problem with slip dependent friction*, Nonlinear Analysis 61 (2005), 135–161.

[3] A. Ochal, *Existence results for evolution hemivariational inequalities of second order*, Nonlinear Analysis 60 (2005), 1369–1391.

*Research supported by the Jagiellonian University Fellowship under contract for AY 2004/2005.

Second order optimality conditions for a control with continuous and bang-bang components

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Abstract

Second order necessary and sufficient optimality conditions for bang-bang control problems in a very general form have been obtained in Milyutin and Osmolovskii (1998). These conditions require the positive (semi)-definiteness of a certain quadratic form on the finite-dimensional critical cone. Using a suitable transformation via a linear matrix ODE, Maurer and Osmolovskii (2003, 2004) have developed numerical methods into test the positive definiteness of the quadratic form. The second order test has been successfully applied to several numerical examples, representing different types of bang-bang control problems. In the present talk we formulate a generalization of these results to optimal control problems with a control variable having two components: a continuous unconstrained control appearing nonlinearly and a bang-bang control appearing linearly and belonging to a convex polyhedron. An important example of a problem of this type is the planar Earth-Mars transfer. The talk is based on a joint work with Helmut Maurer.

Keywords: bang-bang control, critical cone, quadratic form.

References

[1] A.A. Milyutin and N.P. Osmolovskii, Calculus of Variations and Optimal Comtrol, Translations of Mathematical Monographs, Vol. 180, American Mathematical Society, Providence 1998.

[2] H. Maurer and N.P. Osmolovskii, *Second-order sufficient conditions for an extremum in optimal control*, Control and Cybernetics 32 (2003), 555–584.

[3] H. Maurer and N.P. Osmolovskii, *Second order sufficient conditions for time optimal bang-bang control problems*, SIAM J. Control and Optimization 42 (2004), 2239–2263.

Second-order conditions in vector optimization

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Abstract

The aim of this lecture is to present the second-order necessary and sufficient conditions for vector optimization problems. Our results generalize some of previously obtained results ([1, 2, 3, 4]) concerning both scalar and vector optimization problems.

Main result: Let $f : \mathbb{R}^m \mapsto \mathbb{R}^n$ be a function minimized with respect to the pointed closed convex cone C with $\text{int}C \neq \emptyset$, which is continuous near $x \in \mathbb{R}^m$ and ℓ-stable at x.

Assume that $\triangle_l(x) = \{\xi \in C' \cap S_{\mathbb{R}^n}; f^\ell(x;h)(\xi) = 0,\ \forall h \in S_{\mathbb{R}^m}\} \neq \emptyset$, and suppose that for each $h \in \mathbb{R}^m$ one of the following two conditions is satisfied:

(i) $\text{Limsup}_{t\downarrow 0} \frac{f(x+th)-f(x)}{t} \bigcap (-C) \neq \emptyset$,

(ii) $\underline{f}''^{\ell}_P(x;h)(\triangle_l(x)) > 0$.

Then x is isolated mininizer of second-order for f.

For $x \in \mathbb{R}^m$, $h \in S_{\mathbb{R}^m}$, $\xi \in C'$, we define

$$f^\ell(x;h)(\xi) = \liminf_{t\downarrow 0} \frac{\langle \xi, f(x+th) - f(x)\rangle}{t},$$

$$\underline{f}''^{\ell}_P(x;h)(\triangle_l(x)) = \liminf_{t\downarrow 0} \sup_{a\in\triangle_l(x)} \frac{\langle a, f(x+th) - f(x)\rangle - tf^\ell(x;h)(a)}{t^2/2}.$$

Further, Limsup denotes the Kuratowski upper limit set, and by *ℓ-stability* at x, we mean that for some neighbourhood $U \subset X$ and $K > 0$ it holds

$$|f^{\ell}(y;h)(\xi) - f^{\ell}(x;h)(\xi)| \leq K\|y - x\|, \quad \forall y \in U,\ \forall h \in S_{\mathbb{R}^m},\ \forall \xi \in C' \cap S_{\mathbb{R}^n}.$$

Keywords: second-order optimality conditions, vector optimization, isolated minimizer of second-order, peano derivative.

References

[1] D. Bednařík and K. Pastor, *Elimination of strict convergence in optimization*, SIAM J. Control Optim. 43 (3) (2004), 1063–1077.

[2] D. Bednařík and K. Pastor, Using Peano derivative in unconstrained optimization, submitted.

[3] A. Ben-Tal and J. Zowe, *Directional derivatives in nonsmooth optimization*, J. Optim. Theory Appl. 47 (1985), 483–490.

[4] I. Ginchev, A. Guerraggio and M. Rocca, *From scalar to vector optimization*, Appl. Math., in print.

Infinite horizon optimal control problems – Lebesque and Riemann improper integrals

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Abstract

In this paper we consider infinite horizon optimal control problems of the following type:

$$J(x,u) = \int_0^\infty f_0(t, x(t), u(t))\, dt \longrightarrow \text{Min}\,! \tag{1}$$

subject to $x \in W^{1,n}_{p,\nu}(0,\infty)$, $u \in L^r_{p,\nu}(0,\infty)$, satisfying a.e. on $(0,\infty)$
state equations

$$x'(t) = g(t, x(t), u(t))\,,$$

control restrictions

$$u(t) \in U, \quad U \in Comp\,(R^r) \setminus \{\,\emptyset\,\},$$

initial conditions

$$x(0) = x^0.$$

Different criteria of optimality are known for such problems, e. g. the overtaking criterion of v. Weizsäcker (1965), the catching up criterion of Gale (1967) and the sporadically catching up criterion of Galkin (1974). Corresponding to these criteria we prove sufficient conditions for local optimality. Here local optimality is understood in the sense weighted Sobolev spaces $W^{1,n}_{p,\nu}(0,\infty)$, [S], i.e.

$$\|x - x^*\|^2_{W^{1,n}_{p,\nu}(0,\infty)} := \int\limits_0^\infty \{\|(x(t) - x^*(t)\| + \|(x'(t) - {x^*}'(t)\|\}^p \nu(t) dt < \epsilon,\ \epsilon > 0$$

with a density function ν, with $0 < \nu(t) < \infty$, a.e. on $(0,\infty)$ and $\int\limits_0^\infty \nu(t) dt < \infty$.

Some aspects concerning the integral in (1) are essential. We allow both Lebesgue and improper Riemann integrals to appear in integrand of the objective. It can happen, that the integral in (1) as Lebesgue integral does not exist and at the same time the Riemann integral is conditionally convergent. We give applications where the improper Riemann integral repesents the adequate formulation of the problem and discuss analytic properties of the functional.

Keywords: infinite horizon optimal control, weighted Sobolev spaces, Lebesgue and Riemann improper integral.

References

[1] D.A. Carlson, A.B. Haurie and A.A. Leizarowitz, Infinite Horizon Optimal Control, Springer-Verlag, New York, Berlin, 1991.

[2] A. Kufner, Weighted Sobolev Spaces, John Wiley and Sons, Chichester-New York-Brisbane-Toronto-Singapure, 1985.

[3] S. Pickenhain and V. Lykina, Sufficiency conditions for infinite horizon optimal control problems, Proceedings of the 12th French-German-Spanish Conference on Optimization held in Avignon, September 20–24, 2004, to appear in Lectures Notes in Economics and Mathematical Systems, Springer-Verlag.

[4] R. Rosen, Dynamic System Theory in Biology, Wiley-Interscience, 1970.

On existence of solution to degenerate nonlinear optimization problems

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Abstract

We investigate the existence of the solution for the following problem

$$\min \varphi(x), \text{ subject to } F(x) = 0,$$

where $\varphi : X \to \mathbb{R}$, $F : X \to Y$ and X, Y are Banach spaces. The question of existence is considered in a neighborhood of such point x_0 that the Hessian of the Lagrange's function is degenerate. It was obtained approximation for a distance of solution x^* to the initial point x_0.

Keywords: p-regularity, singularity, contracting mapping, non-linear, multivalued, p-factor operator.

References

[1] V.M. Alexeev, V.M. Tihomirov and S.V. Fomin, *Optimal Control*, Consultants Bureau, New York, (1987) Translated from Russian by V.M. Volosov.

[2] A.A. Tret'yakov and J.E. Marsden, *Factor-analysis of nonlinear mappings: p-regularity theory*, Communications on Pure and Applied Analysis 2 (2003) No. 4, 425–445.

[3] A.A. Tret'yakov, *The implicit function theorem in degenerate problems*, Russ. Math. Surv. Vol. 42 (1987), pp. 179–180, 425–445.

Global robust controllability of the triangular integro-differential Volterra systems

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At the first German-Polish Conference on Optimization in Zagan we presented the global controllability for a control system described by a cascade of Volterra integral systems.

Here a solution of the global controllability problem for a bigger class of nonlinear control systems of the Volterra integro-differential equations is given. It is proven that there exists a family of continuous controls that solve the global controllability problem for this class. The contructed controls depend continuously on the initial and the terminal states. It makes possible to prove the global controllability of the uniformly bounded perturbations of these systems under the global Lipschitz condition for the unperturbed system with respect to the states and the controls.

More precisely, the system under consideration has the form

$$\dot{x}(t) = f(t, x(t), u(t)) + \int_{t_0}^{t} g(t, s, x(s), u(s))ds, \quad t_0 \leq t \leq T,$$

where

$$f(t, x, u) = (f_1(t, x_1, x_2), \ldots, f_n(t, x_1, \ldots, x_n, u))^T$$

and

$$g(t, s, x, u) = (g_1(t, s, x_1, x_2), \ldots, g_n(t, s, x_1, \ldots, x_n, u))^T$$

and fulfills certain assumptions.

The paper is available online in the Journal of Mathematical Analysis and Applications since 17 March 2005.

Keywords: nonlinear control, triangular form, global controllability, Volterra integro-differential control systems.

Control of laser surface hardening by a memory efficient approach of second order

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Abstract

Consider a time-dependent optimal control problem, where the state evolution system is described by an ODE. There is a variety of methods for the treatment of such problems. We restrict us to so-called 'indirect methods', namely the Riccati approach for the resulting non-linear BVP with separated BC.

There are many relationships between Multiple Shooting techniques, Riccati approach and a so-called Pantoja method, which describes a computationally efficient stage-wise construction of the Newton direction for the discrete-time optimal control problem.

An efficient implementation of this approach will be presented. Furthermore, previous techniques of checkpointing are extended to a so-called 'nested checkpointing' for multiple transversal's. Some heuristics are introduced, which allow us an efficient construction of nested reversal schedules. We discuss their benefits and compare their results to the optimal schedules computed by exhaustive search techniques. The proposed scheduling schemes as well as Riccati/Pantoja approach are applied to the optimal problem of laser surface hardening of steel.

Finally, we discuss some possible future developments.

Keywords: optimal control, Pantoja/Riccati approach, nested checkpointing.

References

[1] J. Sternberg and A. Griewank, Reduction of storage requirement by checkpointing for time-dependent optimal control problems in ODEs, AD 2004 post-conference special collection.

[2] D. Hömberg and S. Volkwein, *Control of Laser Surface Hardening by a Reduced-Order Approach using Proper Orthogonal Decomposition*, Mathematical and Computer Modelling 38 (2003), 1003–1028.

Remarks on approximations of partially observed control problems

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Abstract

In the talk a number of natural examples of partially observed control problems that arise in mathematics of finance will be considered. We are often in such situation that we observe the prices of assets but are not able to observe (or we observe with a delay) economical factors which influence the dynamics of prices. The purpose is to choose a portfolio maximizing certain utility function or a function of a growth of the portfolio over finite or infinite time horizon. We study a discrete time model since it is closer to real situation (continuous time partially observed model exploit some peculiar properties – observation of the whole trajectory which make the problem almost completely observable). The last part of the talk will be devoted to the construction of nearly optimal strategies. This part will be partially based on the book:

W. Runggaldier, L. Stettner, Approximations of Discrete Time Partially Observed Control Problems, GIARDINI, Pisa 1994.

Keywords: stochastic control, partial observation, mathematics of finance, construction of optimal portfolio strategies.

$\boldsymbol{P}$-factor methods for nonregular inequality-constrained optimization problems

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Abstract

We are considering the nonregular optimization problem

$$\min_{x \in R^n} \phi(x)$$

$$\text{subject to } g_i(x) \leq 0, \; i = \overline{1, m} \tag{1}$$

where gradients $\nabla g_i(x^*)$ are linearly dependent at the solution x^*. Classical methods for solving such a type of optimization problems are not applicable since the Lagrange multiplier λ_0 in the equation

$$\lambda_0 \phi'(x^*) + \lambda_1 g_1'(x^*) + ... + \lambda_m g_m'(x^*) = 0,$$

may be equal zero at the solution point x^*.

We propose to reduce inequality-constrained optimization problem to equality-constrained optimization problem

$$\min_{x \in R^n} \phi(x)$$

$$\text{subject to } f_i(x, y) = g_i(x) + y_i^2 = 0, \qquad i = \overline{1, m} \tag{2}$$

Under assumptions of 2-regularity of the mapping

$$F(x, y) = (f_1(x, y), \ldots, f_m(x, y))^T$$

at the point (x^*, y^*) it followed that must be fulfilled equation

$$f'(x^*) + (F'(x^*, y^*) + P^{\perp} F''(x^*, y^*) h) \, \lambda^T = 0 \tag{3}$$

where $\lambda^T = (\lambda_1, \ldots, \lambda_m)^T$, $h \in \mathrm{Ker} F'(x^*, y^*) \cap \mathrm{Ker}^2 \, P^{\perp} \, F''(x^*, y^*)$ and P is orthoprojector onto $(\mathrm{Im} F'(x^*, y^*))^{\perp}$, and we can apply Newton method.

Keywords: p-regularity, singularity, factor-operator.

References

[1] E. Szczepanik and A.A. Tret'yakov, *The p-regularity theory: constructive analysis of nonlinear optimization problems,* Information Processing: Recent Mathematical Advances in Optimization and Control, pp. 139–160, Paris 2004.

Evolution of structure for direct control optimization

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Abstract

The paper describes new developments in a direct approach to numerical dynamic optimization, called *monotone structural evolution* (MSE). MSE is effective for a large class of nonlinear problems with control and state constraints, including singular cases. Its distinctive feature is that the decision space is systematically recon-structed in the course of optimization, with changing the *control structure*, parameterization and, typically, the number of decision variables. The search for structural changes which lead to rapid improvement of the performance index is based on analysis of the discrepancy between the current approximation of solution and the maximum principle optimality conditions, and continues until these conditions are satisfied with sufficient accuracy. The proper choice of the sequence of decision spaces, utilizing information taken from the adjoint solution, allows the number of decision variables to be kept comparatively small, at least in early stages of optimization. An important property of MSE is that the performance index decreases monotonously during optimization, due to control preservation by the structure changes.

An example in which optimal control is continuous with two types of arcs: boundary and interior is discussed in detail. In every interior arc we approximate the control with the Hermite cubic polynomials. Continuity and smoothness at division points between neighboring interior arcs may be ensured. Derivatives of the performance index w.r.t. control parameters are determined analytically, using adjoint solutions.

The computational experience confirms that the rate of convergence of MSE can be better than in comparable direct methods, and similar to the rate of convergence of indirect algorithms such as multiple shooting or indirect collocation. On the other hand, the area of convergence of MSE is like in other direct algorithms, and larger than in indirect methods.

Bregman-function-based methods for variational inequalities case of non-paramonotone operators

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Abstract

For variational inequalities with multi-valued non-paramonotone operators, characterizing saddle points of Lagrangians associated with convex programming problems in Hilbert spaces, the convergence of an interior point method based on Bregman distance functionals is studied.

The convergence analysis admits a successive approximation of the variational inequality and an inexact treatment of the iterates. Analogous results are proved for discretized complementarity problems with multi-valued monotone operators, which arise for instance in contact problems.

Keywords: complementarity problems, Bregman distances, proximal point methods, contact problems.

[1] A. Kaplan and R. Tichatschke, *Proximal point approach and approximation of variational inequalities*, SIAM J. Control Optim. 39 (2000), 1136–1159.

[2] A. Kaplan and R. Tichatschke, *A general view on proximal point methods to variational inequalities in Hilbert spaces - Iterative regularization and approximation*, J. of Nonlinear and Convex Analysis 2 (2001), 305–332.

[3] A. Kaplan and R. Tichatschke, *Convergence analysis of non-quadratic proximal methods for variational inequalities in Hilbert spaces*, J. of Global Optimization 22 (2002), 119–136.

[4] A. Kaplan and R. Tichatschke, *On inexact generalized proximal methods with a weakened error tolerance criterion*, Optimization 53 (2004), 3–18.

[5] A. Kaplan and R. Tichatschke, *Interior proximal method for variational inequalities: Case of non-paramonotone operators*, Journal of Set-valued Analysis 12 (2004), 357–382.

[6] A. Kaplan and R. Tichatschke, *Extended auxiliary problem principle using Bregman distances*, Optimization 53 (2004), 603–624.

$\boldsymbol{P}$-factor conditions for optimality in extremal problems

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Abstract

We consider optimization problem with equality constraints

$$\text{minimize } \phi(x)$$

$$\text{subject to } F(x) = 0 \tag{1}$$

and the Lagrange function in the form

$$L(x, \lambda_0, \lambda) = \lambda_0 \phi(x) + (F(x))^* \lambda \tag{2}$$

where $F : X \to Y$, $X, Y-$ Banach spaces, $\lambda \in Y^*$.

We are interested in the case when the Lagrange multiplier λ_0 might be equal to zero at the solution point $(x^*, \lambda_0, \lambda^*)$. By applying the apparatus of p-factor operators we have derived new pth order conditions for optimality in the following form

$$P_1 L'(x^*, 0, \lambda^*) + P_2 L''(x^*, 0, \lambda^*)[h] + \ldots + P_p L^{(p)}(x^*, 0, \lambda^*)[h]^{p-1} = 0, \tag{3}$$

where $P_1 = \mathbf{I}, P_2, \ldots, P_p$ some orthoprojectors (see references) and h such element from $(X \times\ \lambda_0\ \times\ Y^*)$ that

$$L'(x^*, 0, \lambda^*)[h] = 0, \ldots, P_p L^{(p)}(x^*, 0, \lambda^*)[h]^p = 0.$$

Keywords: optimality, factor-operator, p-regular.

References

[1] A.A. Tret'yakov and J.E. Marsden, *Factor-analysis of nonlinear mappings: p-regularity theory*, Communications on Pure and Applied Analysis Vol. 2 (2003), 425–445.

Proper orthogonal decomposition surrogate models for nonlinear dynamical systems: error estimates and suboptimal control

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Abstract

Optimal control problems for nonlinear partial differential equations are often hard to tackle numerically so that the need for developing novel techniques emerges. One such technique is given by reduced order methods. Recently the application of reduced-order models to optimal control problems for partial differential equations has received an increasing amount of attention. The reduced-order approach is based on projecting the dynamical system onto subspaces consisting of basis elements that contain characteristics of the expected solution. This is in contrast to, e.g., finite element techniques, where the elements of the subspaces are uncorrelated to the physical properties of the system that they approximate. Proper orthogonal decomposition (POD) provides a method for deriving low order models of dynamical systems. It was successfully used in a variety of fields including signal analysis and pattern recognition, fluid dynamics and coherent structures and more recently in control theory and inverse problems.

In our application we apply POD to derive a Galerkin approximation in the spatial variable, with basis functions corresponding to the solution of the physical system at pre-specified time instances. These are called the snapshots. Due to possible linear dependence or almost linear dependence, the snapshots themselves are not appropriate as a basis. Rather a singular value decomposition (SVD) is carried out and the leading generalized eigenfunctions are chosen as a basis, referred to as the POD basis.

In the talk the POD method and its relation to SVD is described. Furthermore, the snapshot form of POD for abstract parabolic equations is illustrated. We deal with reduced order modeling of nonlinear dynamical systems and present error estimates for reduced order models of a general equation in fluid mechanics obtained by the snapshot POD method. For numerical realization suboptimal control strategies based on POD are shown for optimal open-loop and closed-loop control problems. In particular, error estimation

for linear-quadratic optimal control problems are presented whose discretization is based on POD. Within the talk several numerical examples will illustrate the use of POD in the context of suboptimal control of nonlinear partial differential equations.

The presented results are joined works with Franz Diwoky, Michael Hinze, Dietmar Hömberg, Hannes Müller, Karl Kunisch, Friedemann Leibfritz, and Lei Xie.

References

[1] F. Diwoky and S. Volkwein, *Nonlinear boundary control for the heat equation utilizing proper orthogonal decomposition*, in: Hoffmann, K.-H., Hoppe, R.H.W., Schulz, V., editors, Fast solution of discretized optimization problems, International Series of Numerical Mathematics 138 (2001), 73–87.

[2] M. Hinze and S. Volkwein, Proper orthogonal decomposition surrogate models for nonlinear dynamical systems: error estimates and suboptimal control, To appear in Lecture Notes in Computational Science and Engineering, Vol. 45, 2005.

[3] M. Hinze and S. Volkwein, POD Approximations for optimal control problems govered by linear and semi-linear evolution systems, in preparation.

[4] D. Hömberg and S. Volkwein, *Control of laser surface hardening by a reduced-order approach utilizing proper orthogonal decomposition*, Mathematical and Computer Modelling 38 (2003), 1003–1028.

[5] D. Hömberg and S. Volkwein, *Laser surface hardening using proper orthogonal decomposition for a three-dimensional example*, PAMM Proc. Appl. Math. Mech. 3 (2003), 1–4.

[6] K. Kunisch and S. Volkwein, *Control of Burgers' equation by a reduced order approach using proper orthogonal decomposition*, J. Optimization Theory and Applications 102 (1999), 345–371.

[7] K. Kunisch and S. Volkwein, *Galerkin proper orthogonal decomposition methods for parabolic problems*, Numerische Mathematik 90 (2001) 117–148.

[8] K. Kunisch and S. Volkwein, *Galerkin proper orthogonal decomposition methods for a general equation in fluid dynamics*, SIAM J. Numer. Anal. 40 (2002), 492–515.

[9] K. Kunisch, S. Volkwein and X. Lie, HJB-POD based feedback design for the optimal control of evolution problems, to appear in SIAM J. on Applied Dynamical Systems (2004).

[10] F. Leibfritz and S. Volkwein, Reduced order output feedback control design for PDE systems using proper orthogonal decomposition and nonlinear semidefinite programming, submitted (2003).

[11] F. Leibfritz and S. Volkwein, Numerical feedback controller design for PDE systems using model reduction: techniques and case studies, submitted (2004).

[12] S. Volkwein, Interpretation of proper orthogonal decomposition as singular value decomposition and HJB-based feedback design. Proceedings of the Sixteenth International Symposium on Mathematical Theory of Networks and Systems (MTNS), Leuven, Belgium, July 5–9, 2004

[13] S. Volkwein, *Boundary control of the Burgers equation: optimality conditions and reduced-order approach*, in K.-H. Hoffmann, I. Lasiecka, G. Leugering, J. Sprekels, and F. Tröltzsch, editors, Optimal Control of Complex Structures, International Series of Numerical Mathematics 139 (2001), 267–278.

[14] S. Volkwein, *Optimal control of a phase-field model using the proper orthogonal decomposition*, Zeitschrift für Angewandte Mathematik und Mechanik 81 (2001), 83–97.

Automatic differentiation for equality-constrained optimization

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Abstract

In this talk we discuss the exact computation of derivatives using the technique of Automatic Differentiation (AD) [1]. Complexity estimates for the two basic approaches, i.e. the forward mode and the reverse mode, will be given. This includes for example the "cheap gradient result", i.e. that computational complexity for evaluating the gradient of a scalar-valued function can be bounded above by a computational complexity of the function evaluation multiplied by a small constant. This upper bound allows the usage of exact gradient information for numerous optimization problems.

For vector-valued functions, the reverse mode of AD yields the product of the Jacobian multiplied from the right by a vector. This information can not be approximated, for example, by finite differences. We will present a recent algorithm for equality constrained optimization problems that is mainly based on this adjoint information to approximate the Jacobian of the constraints using quasi-Newton updates. Hence, the possibly very time-consuming forming and factoring of the constraint Jacobian in each optimization step can be avoided. A global convergence result for a more general class of trust-region methods that do not employ the full constraint Jacobian is presented [2]. Finally we discuss some numerical results for PDE-constrained optimization problems.

Keywords: automatic differentiation, quasi-Newton updates, equality constrained optimization.

References

[1] A. Griewank, Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation, SIAM, 2000.

[2] A. Walther, A first-order convergence analysis of trust-region methods with inexact Jacobian, with modified title available as Preprint MATH-WR-01-2005.

Interior point methods and parametric sensitivities in optimal control

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Abstract

Up to now, the two topics of solving optimal control problems with interior point methods and analyzing the stability of optimal solutions in terms of parametric sensitivities have largely been treated independently. In this paper we study the connection between parametric derivatives of interior point approximations and the sensitivities of the optimal control.

For the optimization problem

$$\min_{u \in L_\infty(\Omega)} J(u,p) \quad \text{s.t.} \quad g(u) \geq 0 \quad \text{a.e.}$$

and its interior point regularization

$$\min_{u \in L_\infty(\Omega)} J(u,p) - \mu \int_\Omega \ln(g(u))\, dx$$

we show convergence of the parametric derivatives $u_p(p,\mu)$ of the interior point approximations $u(p,\mu)$ towards the parametric sensitivities $u_p(p)$ of the optimal solution $u(p)$ under suitable convexity and regularity conditions. In particular we derive and discuss the sharp error estimate

$$\|u_p(p,\mu) - u_p(p)\|_{L_q} = \mathcal{O}(\mu^{1/2q}).$$

Numerical examples illustrate the analytical results.

Keywords: interior point method, parametric sensitivities, optimal control.

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